The Structure Function of PSR B0136+57

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Abstract We calculate the theoretical flux density structure function of PSR B0136+57 at 610MHz for both the Kolmogorov spectrum and the super-Kolmogorov spectrum. The theoretical fitted curves for the case of $\beta = 11/3$ is in good agreement with the observation. The result indicates that, the spectrum of the electron density fluctuations is the Kolmogorov spectrum. To some extent, it supports the RISS theory and the extended medium scattering model. It also provides some helpful information of the modification of the model.

1 Introduction

Radio waves emitted by pulsars are dispersed or scattered by the thermal plasma in the interstellar medium (ISM). This kind of propagation effect can be classified into two different scintillations: diffractive and refractive interstellar scintillation (DISS and RISS). Small spacial scale ($10^6 - 10^8$ m) irregularities in the interstellar medium can cause DISS with characteristic time scale of $\sim$ minutes to hours and frequency scale of $\sim$ 100 KHz to 1 MHz. The other kind of scintillation, RISS, is often used to interpret the long-term ($\sim$ days to months) flux variations (Huguenin 1973) mainly at meter wavelengths.

As for the previous work, on the theoretical aspect, the nature and magnitude of the fluctuations such as decorrelation bandwidth, scintillation time scale, and so on have been predicted (Romani et al. 1986). On the observational aspect, the flux modulations and the variations in dynamic spectrum have been measured (Stinebring & Condon 1990; Labrecque et al. 1994; Gupta et al. 1994). Some unusual multiple imaging and extreme scattering have also been observed, owing to refractions by discrete structures in the ISM (Fiedler et al. 1987).

However, the quantitative theoretical calculations of the structure functions have rarely been carried out. In this paper, we calculate the structure function for PSR B0136+57 with the RISS theory and compare the result with the observation.

2 Theoretical Calculation

This pulsar was observed by Stinebring (2000, hereafter S2000). Its structure function has the obvious saturation value and the span of $D(\tau)$ is in a large range of magnitude.

Romani and Blandford developed the RISS theory (Romani et al. 1986; Blandford et al. 1986). For the case of $\beta = 11/3$ and $\beta = 4$, the autocorrelation function is as follows:

(1) β=11/3(Kolmogorov spectrum)

$$\begin{aligned} F(\tau) &= \frac{2Q_*(\frac{\lambda}{2\pi})^4\Gamma(7/6)}{\pi^{3/2}2^{5/6}}H^{-1/3}\int_0^{d_H} dyy^{-1/3} < \theta_s^2(y) >^{-7/6} \\ &\quad (1-\frac{y}{d_H})^2 \exp(-y^2)M(7/6,1,-x), \end{aligned} \tag{1}$$

where

$$Q_* = 3.7 \times 10^{-18} D \; C_{-4} \, (\mathrm{cm}^{-11/3}) \, , \tag{2}$$

$$\theta_s = 2.2 D^{0.6} C_{-4}^{0.6} \lambda^{2.2} \, (\mathrm{mas}) \, . \tag{3}$$

(2) β=4

$$F(\tau) = \frac{Q_* (\frac{\lambda}{2\pi})^4}{\pi^{3/2}} \int_0^{d_H} dy < \Theta_s^2(y) >^{-1} (1 - \frac{y}{d_H})^2 \exp(-y^2) M(1,1,-x), \tag{4}$$

where

$$Q_* = 1.6 \times 10^{-21} D C_{-4} \, (\mathrm{cm}^{-4}), \tag{5}$$

$$\Theta_s = 2.2 D^{0.5} C_{-4}^{0.6} \lambda^2 \, (\mathrm{mas}) \, . \tag{6}$$

The structure function $D(\tau)$ can be written as:

$$D(\tau) = 2 \left[\times F(0) - F(\tau) \right] \, . \tag{7}$$

We calculated the structure function under formulas (1)-(7) using the software Methematical 4.0.

Note that, there are five parameters in the calculation: β, H, D, V, C_{-4}. β is the power law index. D and H are pulsar distance and the distance from the observer to the scattering screen, respectively. V is the transverse velocity of the pulsar relative to the earth and the scattering screen, namely $\boldsymbol{V}_f = \boldsymbol{V}_{\mathrm{pm}} + \boldsymbol{V}_s + \boldsymbol{V}_e$, where $\boldsymbol{V}_{\mathrm{pm}}, \boldsymbol{V}_s, \boldsymbol{V}_e$ represent the proper motion velocity, the velocity of the scattering screen and the earth, respectively. C_{-4} is written as $C_{-4} = \overline{C_N^2}/10^{-4}$, where C_N^2 characterizes the scattering strength and is a function of location(z) in the Galaxy. The selection of the parameters is very important. We calculate the structure function for both $\beta = 11/3$ and $\beta = 4$ by taking the same parameters. The distance 2.9 kpc is derived from DM, based on Taylor & cordes' (1993) electron density model. The fitted distance from the observer to the scattering screen, H, is 1.45 kpc, which means that the scattering screen locates midway between the observer and the pulsar.

The velocity is also a very important parameter. The proper motion velocity of this pulsar is 300 km/s (S2000). We take this value and get a relatively good result for $\beta = 11/3$. As for the scattering strength, the fitted value of $10^{1.5} \; m^{-20/3}$ is equal to the observed value from S2000.

Finally, the fitted results for the two cases are quite different. For $\beta = 11/3$, the calculated result is consistent with the observation very well. However, for $\beta = 4$, the theoretical curve departs very far from the observation. So there is a strong evidence that the spectrum of the electron density irregularities along the line of sight of this pulsar is the Kolmogorov spectrum, not the super-Kolmogorov spectrum, which is in agreement with Stinebring' opinion (S2000).

3 Analysis of the Parameters

In figure 2, we show the effects of the parameters on the structure function. In each figure, the solid line is the best fitted curve with the parameters: $\beta = 11/3$; $V = 300$ km/s; $D = 2.9$ kpc; $H = 1.45$ kpc; $C_{-4} = 10^{1.5} \, m^{-20/3}$. Keeping the other four parameters invariable, we change one parameter during calculation. Figure 2a-2d show the variations of H, V, C_{-4} and D, respectively.

As for the velocity, it does not affect the saturation value, but it changes the structure function of the structure region. The structure function of the structure region increases with

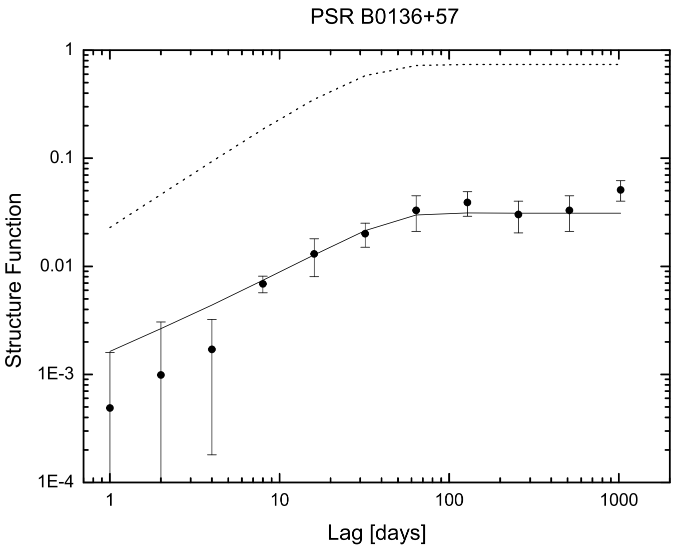

Fig. 1 Observed (the solid dots) and theoretical flux density structure functions of pulsar PSRB0136+57. Solid and dotted lines represent the structure functions for the case of $\beta = 11/3$ and $\beta = 4$, respectively.

the velocity. Seen from figure 2b, the proper motion velocity value of 300 km/s may be larger. The smaller velocity is more realistic and better. For example, the dotted curve adopting the velocity value of 200 km/s is better consistent with the observation. One reason is that, the scintillation velocity is obviously different from the proper motion velocity. Maybe the velocity of the scattering screen is too high, thus leading to the lower transverse velocity. The other likely interpretation is the measurement error of the proper motion velocity.

Figure 2c shows that, the theoretical curve descends with the scattering strength. Talor & Cordes (1993) thought that, for most known pulsars, the new electron density model provides distance estimates accurate to $\sim$25% or better. So we modulated the distance to calculate the structure function, and found that the curve descends with the distance.

The effect of H on structure function is shown in figure 2a. When it increases, the structure function of the structure region decreases, while the saturation value increases. But when the scattering screen is near to the pulsar, it has little effect on the structure function.

4 Results and Conclusions

The fitted curves and comparison with the observation are shown in figure 1. There are three important quantities to characterize the structure functions of the pulsars: m, τ_r, p. m, the modulation index, denotes the flux variation intensity. We calculate it by this formula: $m = \sqrt{D_\infty/2}$, where $D\infty$ is the saturation value. τ_r expresses the value of refractive time scale and is defined as the time lag at which the structure function reaches one-half its saturation value. The theoretical time scale is calculated by $D(\tau_r) = D_\infty/2$. The relationship $D(\tau) \propto \tau^p$ is used to calculate p.

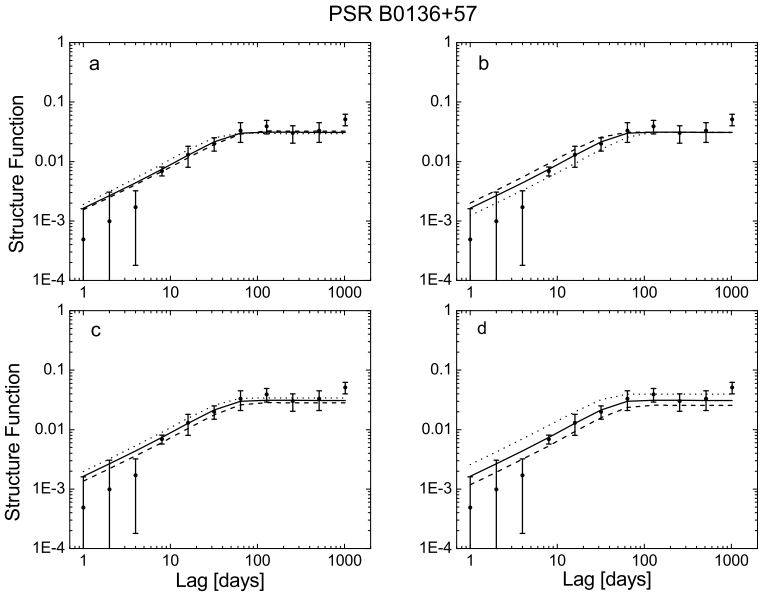

Fig. 2 Structure function vs time lag on the parameters of H, V, C_{-4} and D for PSR B0136+57. Solid dots and lines are the observational data (S2000) and the best fitted curves, respectively. (a) The dashed, solid and dotted lines represent $H = 2\,\mathrm{kpc}, 1.45\,\mathrm{kpc}$ and $1\,\mathrm{kpc}$, respectively. (b)The dashed, solid and dotted lines represent $V = 400$ km/s, 300 km/s and 200 km/s, respectively. (c)The dashed, solid and dotted lines represent $C_{-4} = 10^{1.6}$, $10^{1.5}$ and $10^{1.4}$, respectively. (d)The dashed, solid and dotted lines represent $D = 3.6\,\mathrm{kpc}$, $2.9\,\mathrm{kpc}$ and $2.2\,\mathrm{kpc}$, respectively.

The theoretical modulation index value of 0.12 is almost equal to the observed value of 0.15 ± 0.02. The calculated time scale 21 days is shorter than the observed value of 26 days, and the theoretical slope 0.74 is different from the observed value of 1.4. However, the fitting curve is still in the range of the error bars, so the calculated result is consistent with the observation. It proves that, to some extent, the RISS theory can account for the long-term flux variation of this pulsar. Maybe we can adopt the "two component" model (Qian et al. 1995) to get a better result.

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