# Multi-Pulsar Cross-Correlation Method for Detecting Cosmic Gravitational Waves

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**Abstract** The method of cross-correlation of timing residuals of many pulsars to detect the cosmic gravitational waves is studied. An analytic expression has been derived for the mean squared cross-correlation residuals in terms of the energy density of cosmic gravitational waves for a general multi-pulsar case.

The existence of the gravitational waves predicted in the theory of General Relativity has been observed from the binary pulsars PSR B1913+16 indirectly long time ago. Over the last two decades astronomers have been trying to use millisecond pulsars as a highly accurate timing tool to detect directly the cosmic gravitational waves that may have left over from the early Universe. These studies are important not only in the detection of cosmic gravitational waves itself, but also in determining the matter content and the structure formation of the Universe. In this paper we study physical concepts involved in the timing residuals of the radio signals from millisecond pulsars and derive an analytic expression of the mean squared cross-correlation residuals in terms of the energy density of cosmic gravitatioanl waves for the multi-pulsar observations. We also discuss briefly the advantage of using the multi-pulsar cross-correlation residuals.

The method using the single-pulsar radio signal residuals to detect the gravitational waves has long been well known. In the observations of the radio signals from a millisecond pulsar both the observer and the pulsar may be subject to the influences of the comic gravitational waves. The formula for the energy density of the cosmic gravitational waves in terms of the auto-correlation of the signal residuals from a single-pulsar has been given by Detweiler (1979). Assuming that the cosmic gravitational wave spectrum is flat with a constant energy density within the frequency interval $[\frac{1}{2}f,\ \frac{3}{2}f]$ and zero outside, one has the auto-correlation of timing residuals

$$< R_0^2(t) >= \frac{208G\rho}{243\pi^3 f^4}\,, \tag{1}$$

where $R_0(t)$ is the residual of the pulsar signals, $G$ is the gravitational constant, $f$ is central frequency of the cosmic gravitational waves, and $\rho$ is the energy density of the gravitational waves. By measuring $R_0$, we can obtain an upper limit of $\rho$. This method has also been used by Thorsett & Dewey (1996) to process the seven-year accumulated data of the two single millisecond pulsars PSR B1937+12 and PSR B1855+09. The upper limit has been obtained as being $\rho/\rho_c \le 10^{-8}h^{-2}$ of the energy density of the cosmic gravitational waves, where $\rho_c$ is the critical energy density.

Just like in any astronomical observations, there is always random noise accompanying the radio signals from pulsars. This will mix up with the cosmic gravitational waves. By using the multi-pulsar cross-correlation method one can reduce the level of noise and yield a better detection of a possible common gravitational wave signal (Hellings & Downs 1983).

Usually two effects will cause a shift in the received radio signal timing from pulsars. One is the gravitational waves and the other is the noise. So the fractional frequency shift in the signal of the pulsar $i$ may be written as

$$\frac{\Delta\nu_i(t)}{\nu_i} = \alpha_i h(t) + N_i(t)\,, \tag{2}$$

where $N_i(t)$ represents all random noise sources unique to the pulsar $i$ (including the random gravitational waves passing through the pulsar $i$), $h(t)$ is the gravitational wave amplitude at the Earth, $\alpha_i$ gives the angle factor $\frac{1}{2}\cos 2\phi(1-\cos\theta)$ for the $i$th pulsar, $\theta$ is the angle between Earth-pulsar distance and the wave propagation direction ($z$-axis), and $\phi$ is the angel between a principle polarization vector of the wave and the projection of the pulsar position on the transverse plane ($x-y$ plane). We notice that the random cosmic gravitational waves occurring at the two pulsars have been absorbed in the noise term in Eq(2) and will be cancelled out in the cross-correlation in the following. The residual of $i$-th pulsar by definition is $R_i(t) = \int_0^t \frac{\Delta\nu_i}{\nu_i}dt'$. We define the Two-Pulsar Cross-Correlation Function (TCF):

$$\begin{gathered} R_{ij}^2 \equiv \tfrac{1}{2} < R_i(t)R_j(t+\tau) + R_i(t)R_j(t-\tau) > \\ = \tfrac{1}{2T}\int_{-T/2}^{T/2}[\oint d\Omega[\int_0^t\int_0^{t+\tau} + \int_0^t\int_0^{t-\tau}](\alpha_i h\alpha_j h + \alpha_i hN_j + \alpha_j hN_i + N_iN_j)dt'dt'']dt\,, \end{gathered} \tag{3}$$

where $T$ is the time duration the pulsars are monitored, the terms containing $N_i$ and/or $N_j$ are the noise terms. Since the noise is uncorrelated, the noise terms go to zero after integration. It is usually assumed that $h(t)$ is due to an isotropic background and is continuous and stochastic, so that the term of $< hh >$ can be moved outside the angle integration, and we obtain, after some calculation

$$R_{ij}^2(t) \approx \frac{104G\rho}{81\pi^3 f^4}\alpha_{ij}\,, \tag{4}$$

where the mean angle factor $\alpha_{ij}$ is given by

$$\alpha_{ij} = \frac{1}{4\pi}\oint \alpha_i\alpha_j d\Omega = \frac{1-\cos\gamma_{ij}}{2}\ln(\frac{1-\cos\gamma_{ij}}{2}) - \frac{1-\cos\gamma_{ij}}{12} + \frac{1}{3}\,. \tag{5}$$

For the multi-pulsar observations we define the Multi-Pulsar Cross-Correlation Function (MCF) as

$$R_{12\ldots n}^2 \equiv \sum_{i\neq j}^{n} R_{ij}^2\,, \tag{6}$$

where $R_{ij}^2$ is given Eq.(3). Similar calculation yields the result

$$R_{12\ldots n}^2 = \frac{104G\rho}{81\pi^3 f^4}\sum_{i\neq j}^{n}\alpha_{ij}\,. \tag{7}$$

In this formula $R_{12\ldots n}^2$ and $\sum\alpha_{ij}$ are obtained from the measurement.

The method of multi-pulsar cross-correlation that we presented above is much better than that of single pulsar auto-correlation in order to detect the cosmic gravitational waves. The more pulsars, the better. This is because the noise occurring at each of the multi-pulsars under consideration are cancelled out in the formulation of cross-correlation. Therefore, we advocate a multi-pulsar monitoring project with regard to detection of gravitational waves whenever it is possible.

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## References

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