A New Method of Estimation for Magnetic Inclination Angle of Pulsars

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Abstract Several methods to estimate magnetic inclination angle need good polarization measurement. We propose a new method for different classes of pulsars based on the classification of profile and the distribution of Q parameter. The method is applicable to pulsars without good linear polarization measurement but with good classification. We calculated the magnetic inclination angles for 14 triple and multiple pulsars.

1 Introduction

Magnetic inclination angle α is a very important parameter for pulsar. Several methods have been proposed to calculate the magnetic inclination angle in the frame of magnetic pole model, but there is no perfect method for most pulsars now.

Based on the geometrical magnetic pole model, one obtains two equations:

$$\cos\triangle\phi = \frac{\cos\rho - \cos\alpha\cos\theta}{\sin\alpha\sin\theta}\,, \tag{1}$$

$$(\frac{d\psi}{d\phi})_{\max} = \frac{\sin\alpha}{\sin(\theta-\alpha)}\,, \tag{2}$$

where θ is the angle between the rotation axis and the observer's line of sight, $2\triangle\phi$ is the total pulse width in longitude, ρ is the angular radius of emission beam and α is the angle between the rotation axis and magnetic axis. The maximum rate of the position angle swing could be measured from high quality polarization. Since $\triangle\phi$ and $(\frac{d\psi}{d\phi})_{\max}$ could be measured from observation, only 3 parameters need to be determined, so we need another equation besides eqs. (1) and (2).

Kuzmin(1984) first proposed the equation of the relation between the emission beam radius ρ and the period P, which reads:

$$\rho = 5.0P^{-1/2}, \tag{3}$$

Combing this equation with (1) and (2), he obtained the angle α. Lyne and Manchester (LM, 1988) revised the $\rho - P$ relation:

$$\rho = 6.5P^{-1/3}\,. \tag{4}$$

They calculated α for about 100 pulsars. Though many authors did not agree with their relation, the results of LM were still used widely.

The statistic relation of $\rho - P$ has been debated for many years. Some authors suggested that the index should be -0.5. After analyzing a large body of profile data, Rankin (1993) proposed the core-double-cone model, which suggests that the emission beam consist of a central core and two outercones. The $\rho - P$ relation given by Rankin is:

$$\rho_{\rm outcone} = 5^o.75P^{-1/2}, \tag{5}$$

This method needs high quality linear polarization angle swing curve to calculate the maximum rate, however, it is difficult to be obtained. Till now not more than 1/4 of the pulsars have reliable measurements.

For the triple(T) and the five-component pulsars with core, the line of sight must travel across the core component. So it is near the center of the emission beam. Rankin presumed that the impact angle β equal 0, then she gave:

$$W_{50} = 2^o.45P^{-1/2}/\sin\alpha, \tag{6}$$

She (1991) also used this equation to estimate the α for those pulsars with core component. But in fact it is difficult to measure the half width W_{50} of the core because it is often superposed by the nearby components. Here we develop a new method which is based on the classification of integrated pulse profile and the distribution of Q for different class pulsars.

2 The Relation Curve of Magnetic Inclination Angle α and Q

Wu et al. (1986) defined the parameter Q:

$$Q = \frac{|\theta - \alpha|}{\rho}, \tag{7}$$

The parameter is used to describe the trajectory of the line of sight across the emission cone. It is different from β, the impact angle, which can not represent the position of the sight line traveling across the emission beam. However, the Q is the ratio of β to ρ. A bigger Q represents the sight line sweeps across the outer emission beam; and a smaller one the sight line nearer to the magnetic axis. When $Q = 0$, the sight line travels across the center of the beam. The range of Q is: $0 \leq Q \leq 1$.

Rankin (1993) proposed the core-double-cone model and she classified the pulse profile into different types as M(multiple), T(triple), D(double), Sd(conal single), St (core single), cQ(conal quartre) and cT(conal triple). According to the structure of the emission beam, the different types will have different Q values. Apparently for the M and T pulsars, since the sight line across the core component, Q should be small. For cQ, cT and D pulsars, the value of Q should be medium, and for the Sd, Q should be the largest. So it is possible to give the Q for different class.

On the basis of the classification of Rankin we pick out the pulsars with clear profile classification and good polarization data, then calculate their values of Q using the maximum rate of position angle swing $(\frac{d\psi}{d\phi})_{\max}$ and the relation between α and ρ. That is, we can obtain the Q from equation (1) and (2), and then using these values we can calculate the average Q for different classes. The average value of M and T pulsars is $Q_M = 0.29 \pm 0.12$.

3 The Relation Between α and Q

From equation (1) and (7), we can get:

$$cos(2\alpha \pm \rho Q) = \cos\rho Q - 2\frac{\sin\rho(1+Q)/2\sin\rho(1-Q)/2}{\sin^2\triangle\phi/2}, \begin{cases} \theta > \alpha, + \\ \theta < \alpha, - \end{cases}, \tag{8}$$

where, $\theta < \alpha$ means that the sight-line's trajectory is between the magnetic and rotation axes, and $\theta > \alpha$ means the outer. The sign in the equation (8) is decided by the sign of $(\frac{d\psi}{d\phi})_{\max}$:

$$(\frac{d\psi}{d\phi})_{\max} > 0 \qquad \theta > \alpha, \tag{9}$$

$$(\frac{d\psi}{d\phi})_{\max} < 0 \qquad \theta < \alpha\,. \tag{10}$$

The relation between α and Q can be obtained from equation (8). For the same value of Q, the magnetic inclination (α) for $\theta < \alpha$ is larger than that for $\theta > \alpha$. When $Q = 0$, the α has the same value of $\arcsin(\frac{\sin(\rho/2)}{\sin\triangle\phi/2})$ for the two cases and then decreases with Q. When $\theta > \alpha$, α limits to 0^o; and when $\theta < \alpha$, α limits to ρ. According to the increase of $\triangle\phi$, the α falls quickly at first, but when $\triangle\phi/\rho$ becomes larger, the magnetic inclination angle will not be sensitive with Q. That is, the α will vary slowly with Q.

4 New Method to Measure the Magnetic Inclination Angle

After we get the average Q value of different classes, we can regard the mean value as the real value of this type pulsar. Then using equation (8), Q value, the $\rho - p$ relation and the direction of the polarization position swing, we can obtain the magnetic inclination angle. Even if the direction of the swing is not available, we can also get two values for $\theta < \alpha$ and $\theta > \alpha$ separately. Since the distinction of these two values is so little, we take the average of them as the inclination angle of this pulsar. The error of α results from the dispersion of Q. We know that the less the Q, the less the error; the larger $\triangle\phi/\rho$, the less the error. The Q of the triple and multiple is the least, so we choose 14 pulsars of triple and multiple. In table 1 we list their magnetic inclination angles estimated by our new method. Their Q we used is 0.29 and the $\triangle\phi$ of mostly pulsars are much larger than their ρ. So the estimation error is very small.

Table 1 The result of the inclination angle α

PSR B	Class	$P(s)$	$\triangle\phi(^o)$	$\rho(^o)$	$\frac{\triangle\psi}{\triangle\phi}$	$\alpha(^o)$
$0011+47$	T	1.24	15.8	5.2	$-$	17.3
$0059+65$	T	1.68	10.9	4.4	$-$	21.8
$0727-18$	T	0.51	10.5	8.1	$+$	50.3
$1607-13$	T	1.02	17.7	5.7	$+$	19.7
$1657-13$	T	0.64	19.0	7.2	$-$	19.7
$1738-08$	M	2.04	10.1	4	$+$	23.6
$1746-30$	T	0.61	65.4	7.4	$-$	4.8
$1756-22$	T	0.46	7.6	8.5	$-$	42
$1851-14$	T	1.15	20.4	5.4	$-$	18.7
$1857-28$	M	0.61	18.9	7.4	$-$	28.8
$1944+17$	T	0.44	10.3	8.7	$+$	35.9
$2000+40$	M	0.91	13.7	6	$+$	20.2
$2154+40$	T	1.53	13.0	4.6	$+$	15.5
$2319+60$	T	2.26	26.1	3.8	$-$	7.2

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