

# Pulsar Timing and Turbulence in the Interstellar Plasma

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How does turbulence in the ISM affect pulsar observations,  
and what can we learn about it from pulsar observations?

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All pulsar observations must be corrected for the effects of propagation through the interstellar plasma (IISM).

So all pulsar observations must include a measurement of the IISM and these can be used to study the IISM.

What can we learn about the IISM? It is undoubtedly turbulent and we would like to know:

- the scales at which energy is introduced and dissipated
- the relation to galactic structure and magnetic field
- the character of the micro-structure (isotropy, homogeneity)

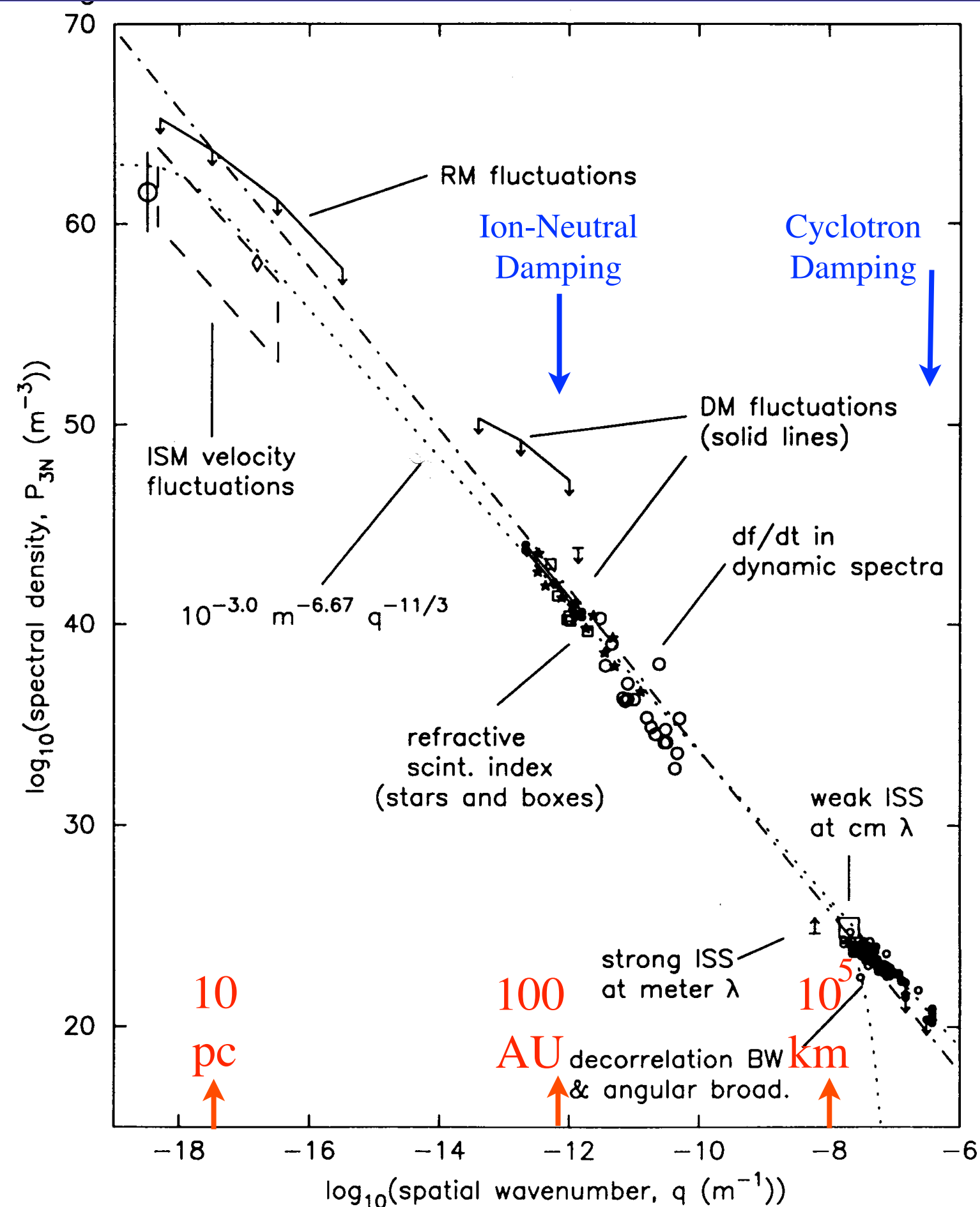
# The Great Power-Law in the Sky

(Armstrong, Rickett, & Spangler, ApJ, 1995)

This is a synthesis of the spatial power spectrum of electron density from a wide variety of measurements (mostly pulsars)

Underlying the plot are the assumptions of homogeneity and isotropy.

We are no longer confident that either assumption is valid.



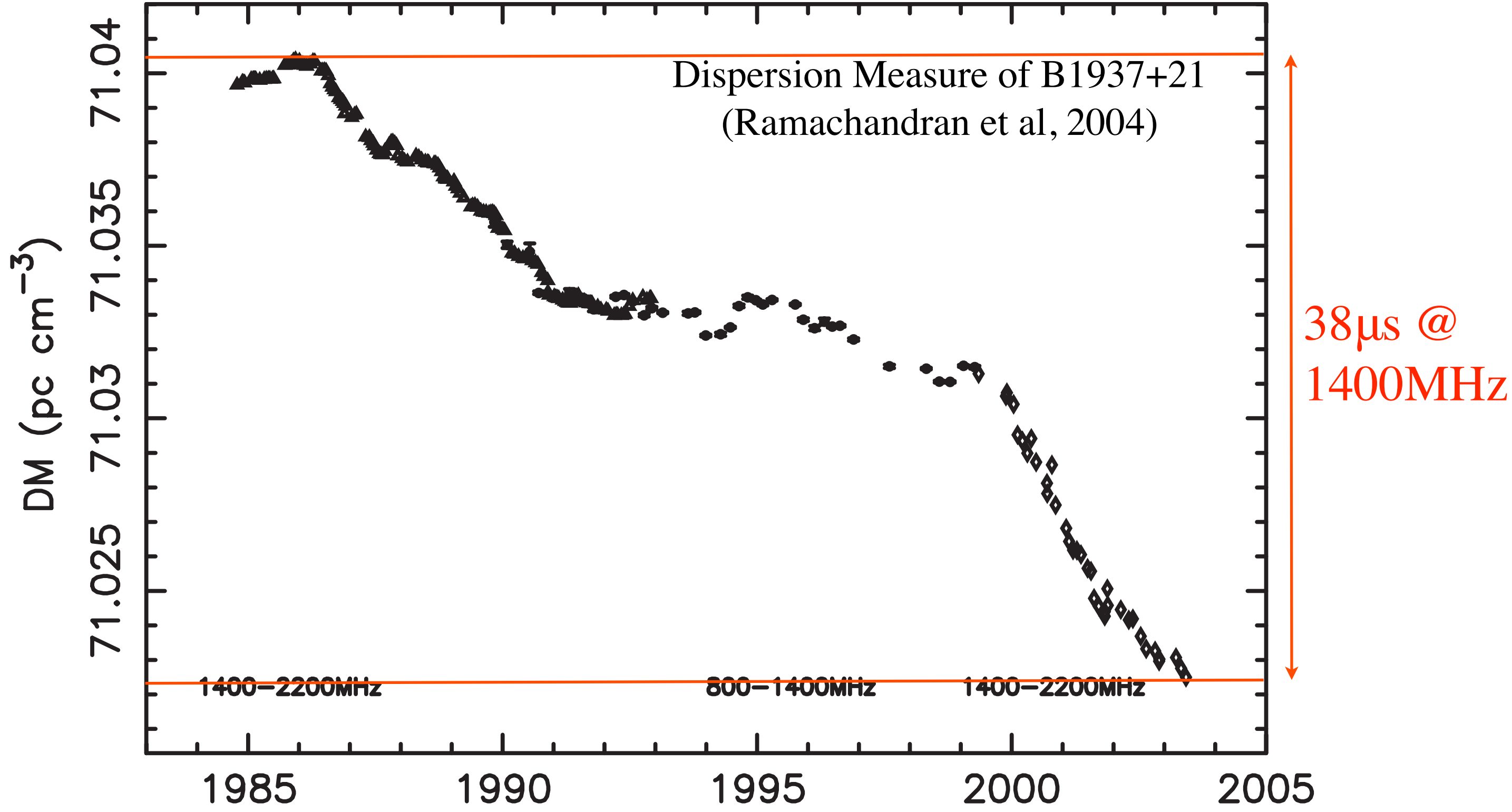
# Primary Observables

All pulsar observations are made with spectrometers, because the *group delay*  $\tau_g$  due to the IISM is large and dispersive. It is  $\tau_g = \text{DM} / 2.41 \times 10^{-4} \nu^2$  where DM is the path-integrated electron density ( $\text{pc cm}^{-3}$ ).  $\tau_g$  must be measured accurately and corrected.

The spectrometer provides a “*dynamic spectrum*” of apparent flux vs time and frequency. The intensity fluctuates due to scattering due to electron density fluctuations in the IISM.

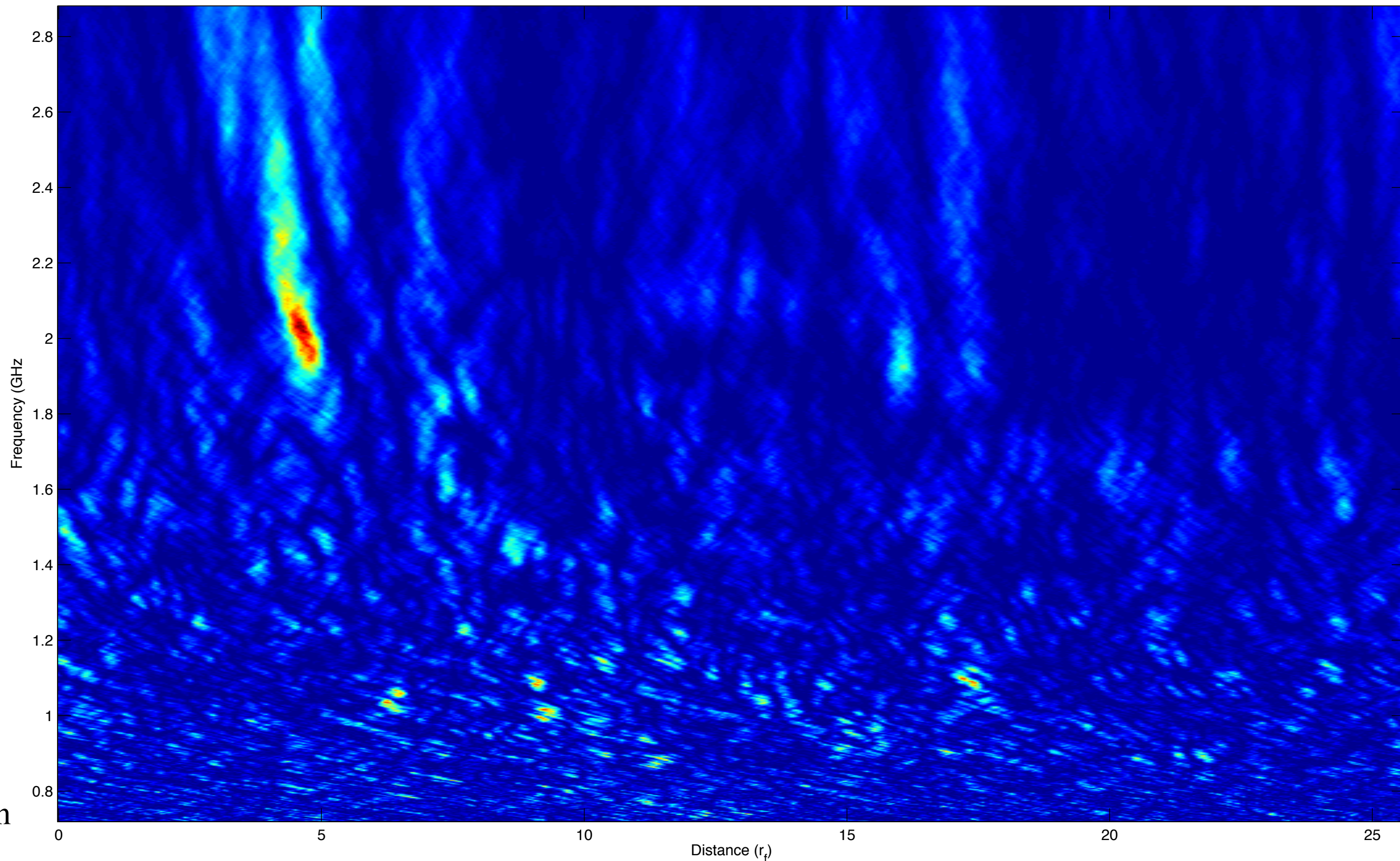
These intensity “*scintillations*” have a characteristic *time scale*  $\tau_{\text{diff}}$  and *frequency scale*  $\nu_{\text{diff}}$ . They also change  $\tau_g$

The IISM is turbulent, so the DM varies with both position and time



# A typical PTA pulsar would look like this if we could observe from 700 MHz to 3 GHz

Weak  
Scintillation



Strong  
Scintillation

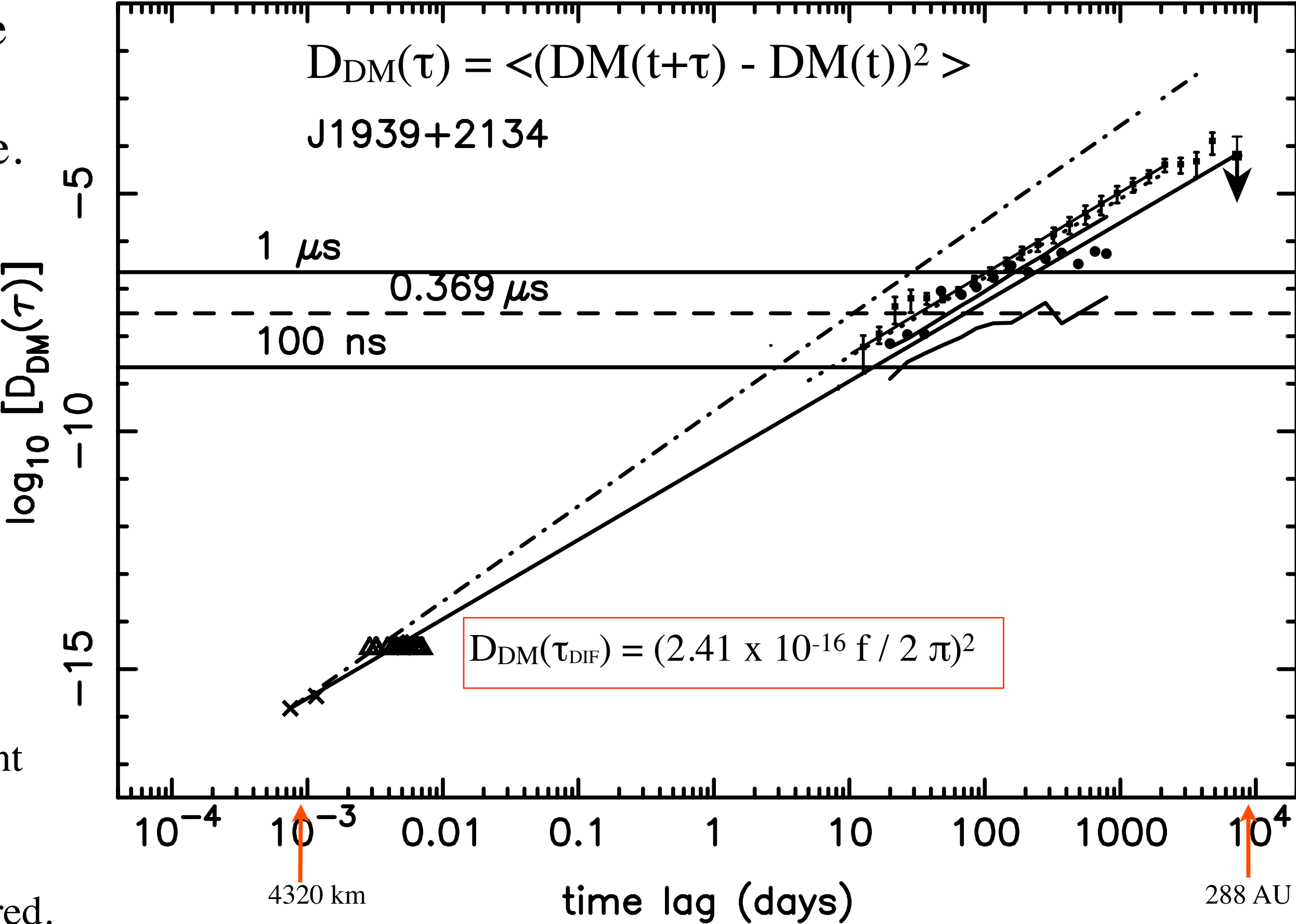
A day or two

We can use the DM(t) and  $\tau_{\text{diff}}$  measurements (You et al., 2006) to estimate the power spectrum of the turbulence in the direction of a single pulsar, avoiding the problem of combining observations of many pulsars.

However the natural statistic to estimate is not the power spectrum but the *structure function*  $D(\tau)$ . This statistic is almost an inverse of the power spectrum. It provides a distribution of energy vs time scale, rather than vs frequency.

A turbulent power spectrum is  $P(f) \propto f^{-8/3}$  whereas the corresponding structure function is  $D(\tau) \propto \tau^{5/3}$

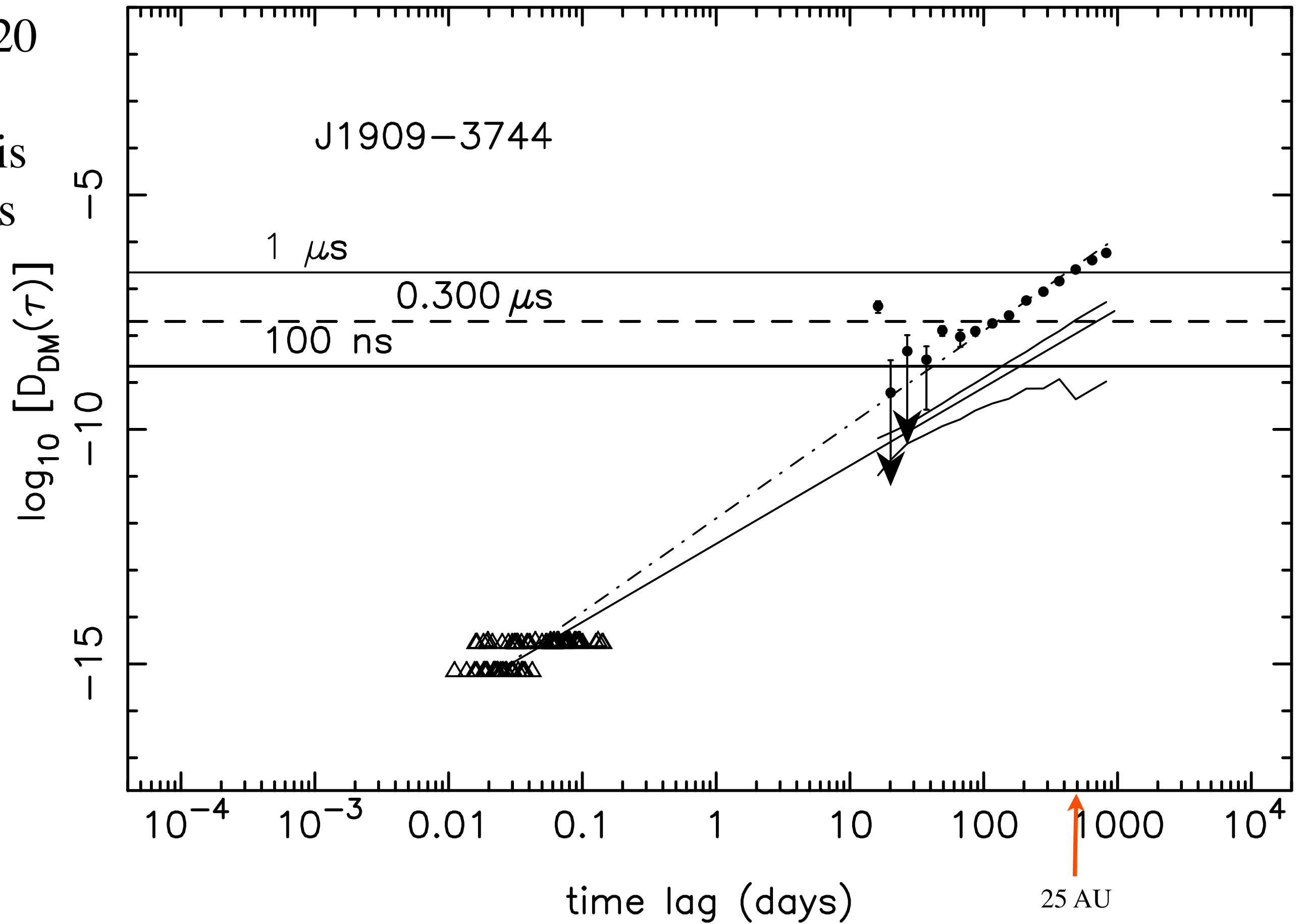
Most of the time the spectral exponent is Kolmogorov as here.



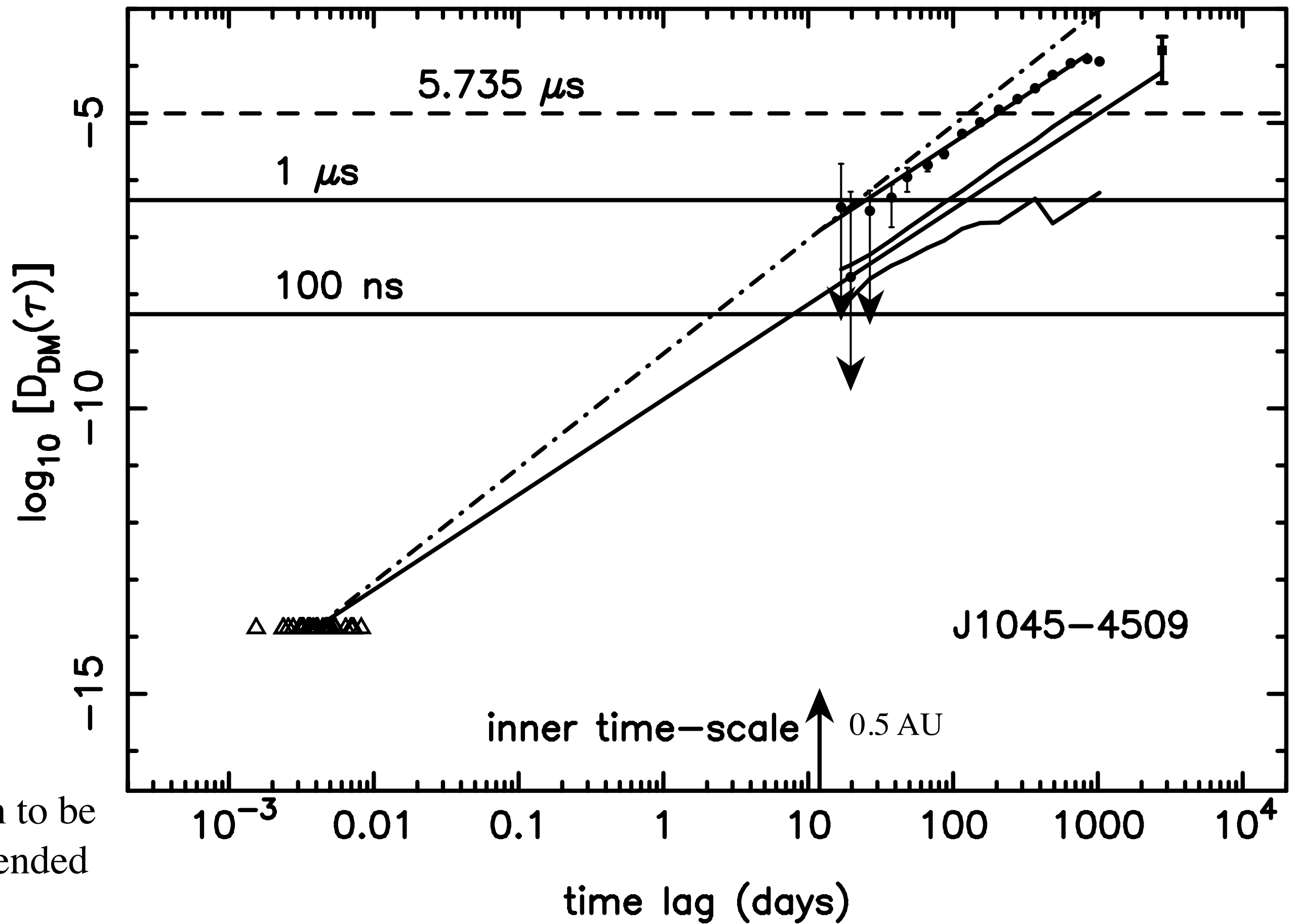
Both white measurement errors and estimation errors due to finite data length must be considered.



However for 2 of 20 PPTA pulsars the spectral exponent is not Kolmogorov as here.



and here.



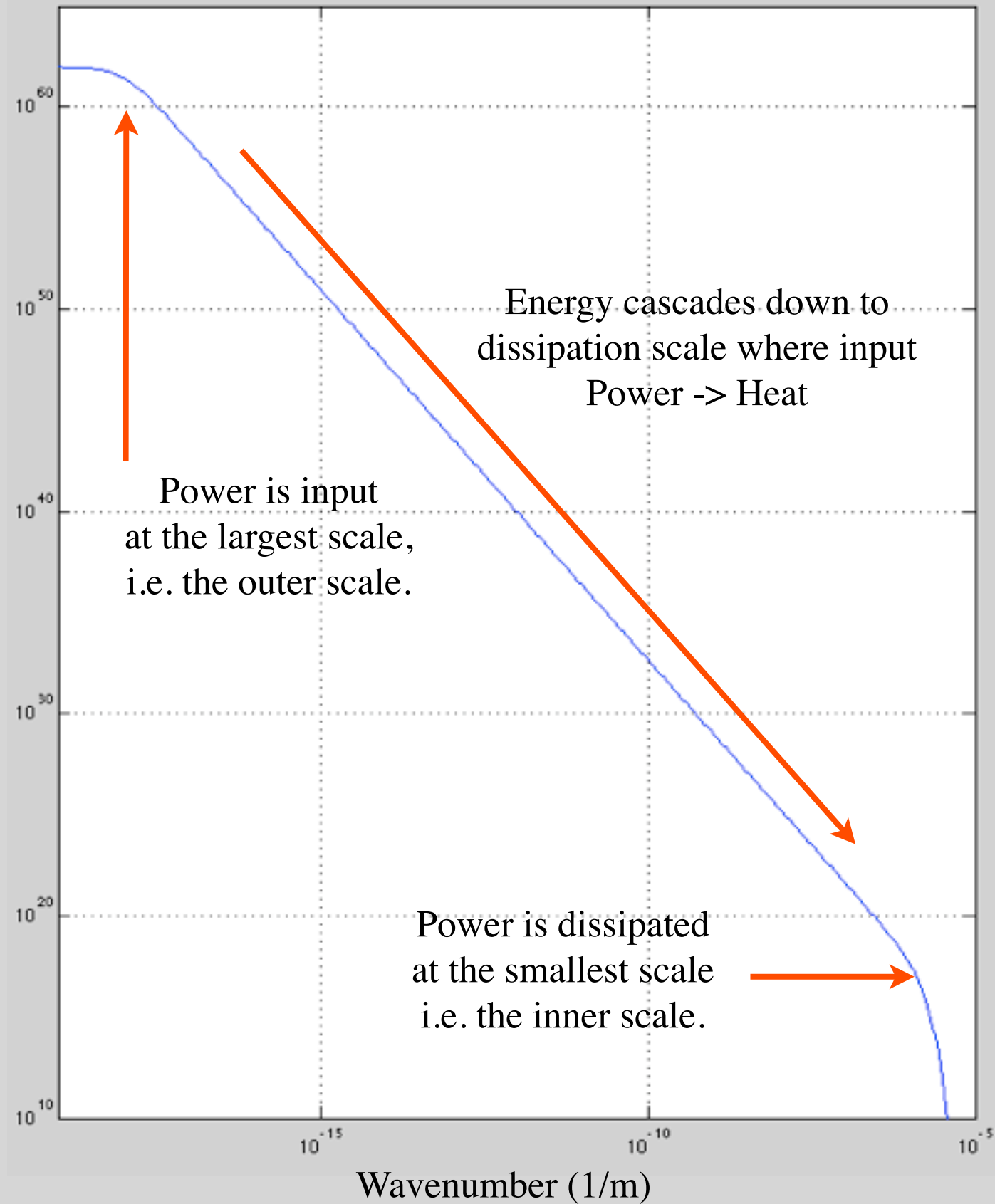
These estimates remain to be confirmed with the extended PPTA observations.

The non-Kolmogorov cases are extremely interesting because:

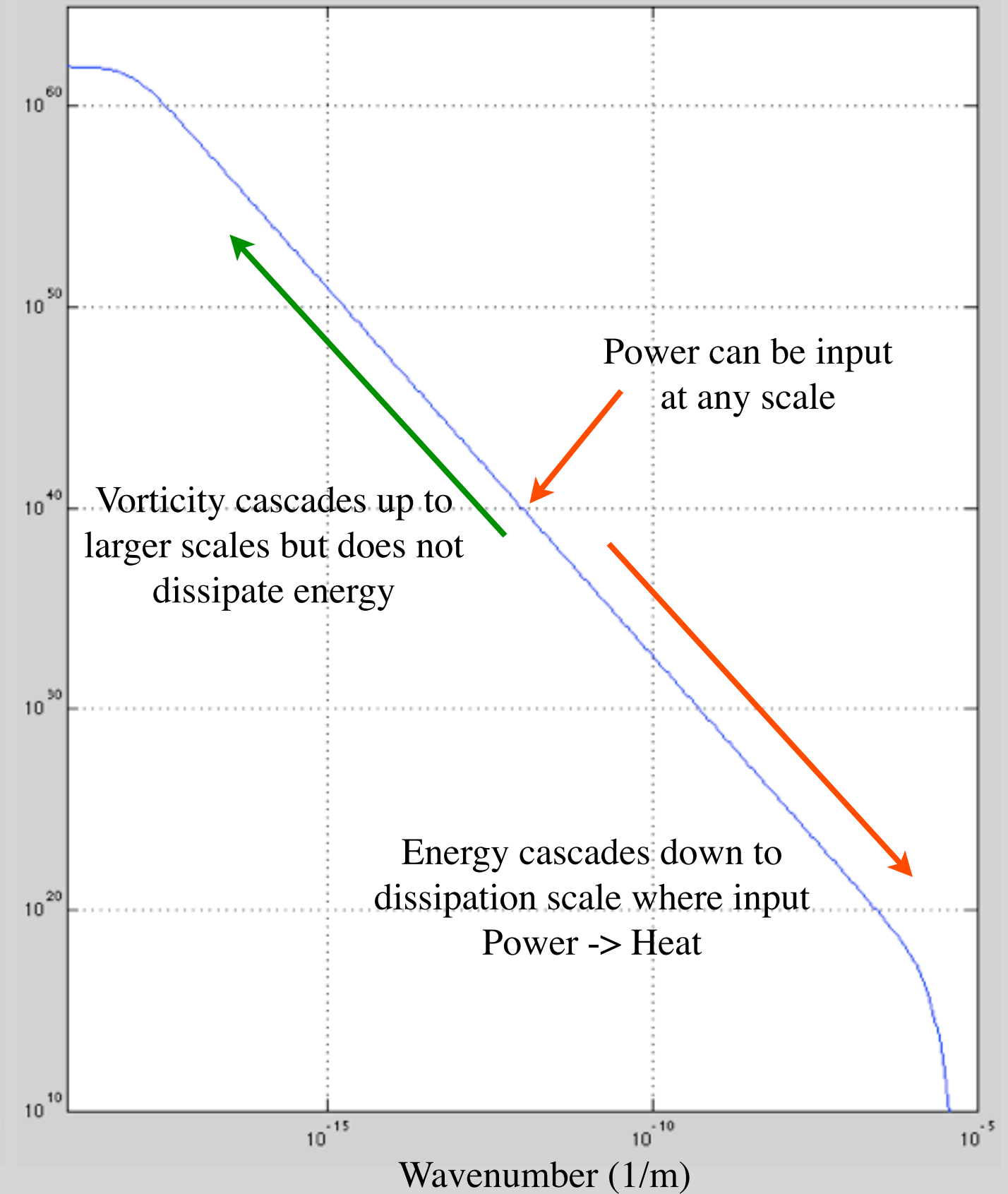
- they would be the first observation indicating dissipation at such a large scale,
- such turbulence would scatter cosmic rays differently,
- such spectra might imply an Alfvénic band-gap due to ion-neutral collisions, if so it would be the first confirmation that this mechanism actually exists,
- if a bandgap exists, it implies that the turbulence is Alfvénic, and it provides a method for studying the turbulent cascade!

This is a very promising area for future research!

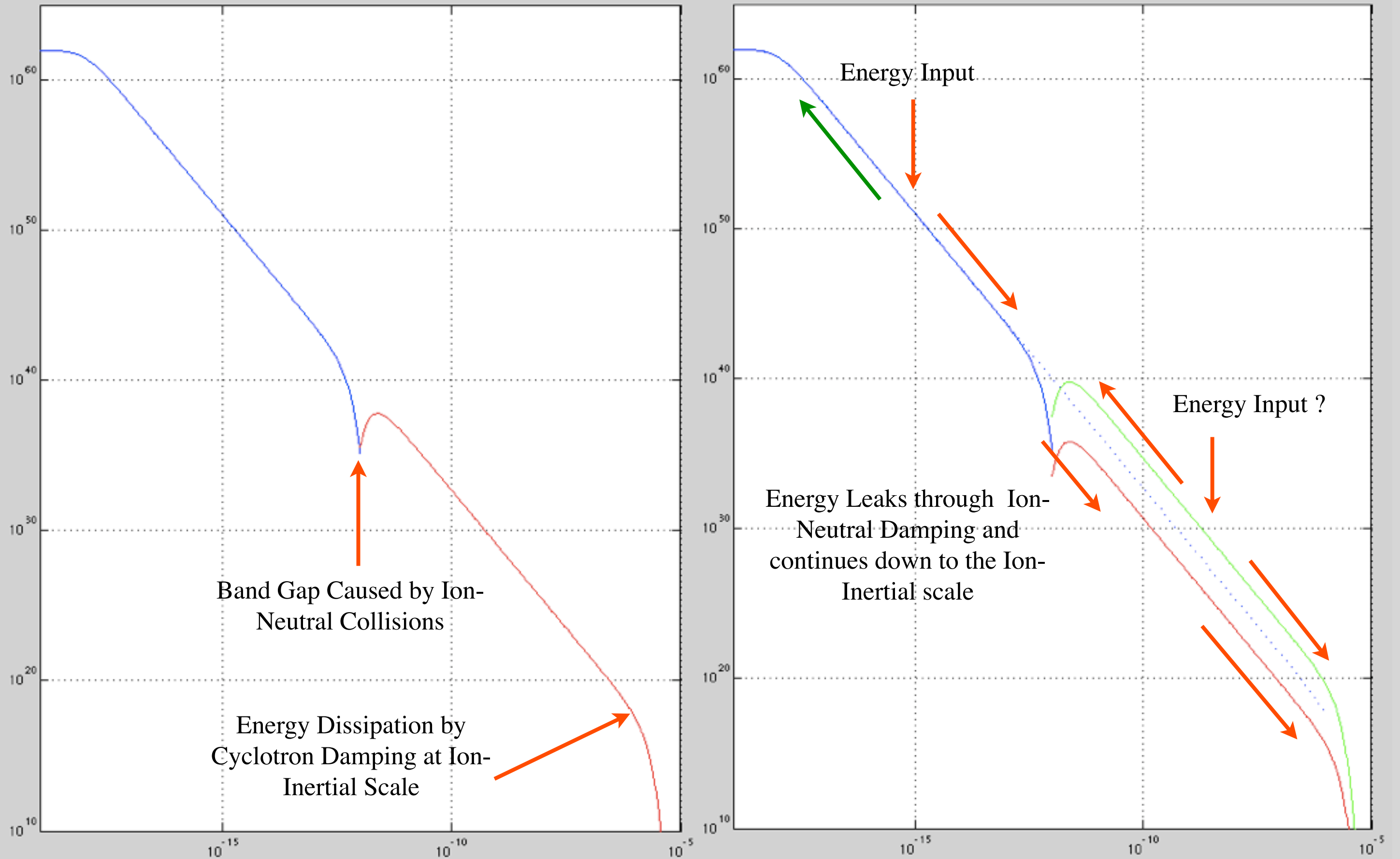
# Turbulence - the naive view



# A better, but incomplete, view



# Effect of Ion-Neutral Collisions on Alfvénic Turbulence

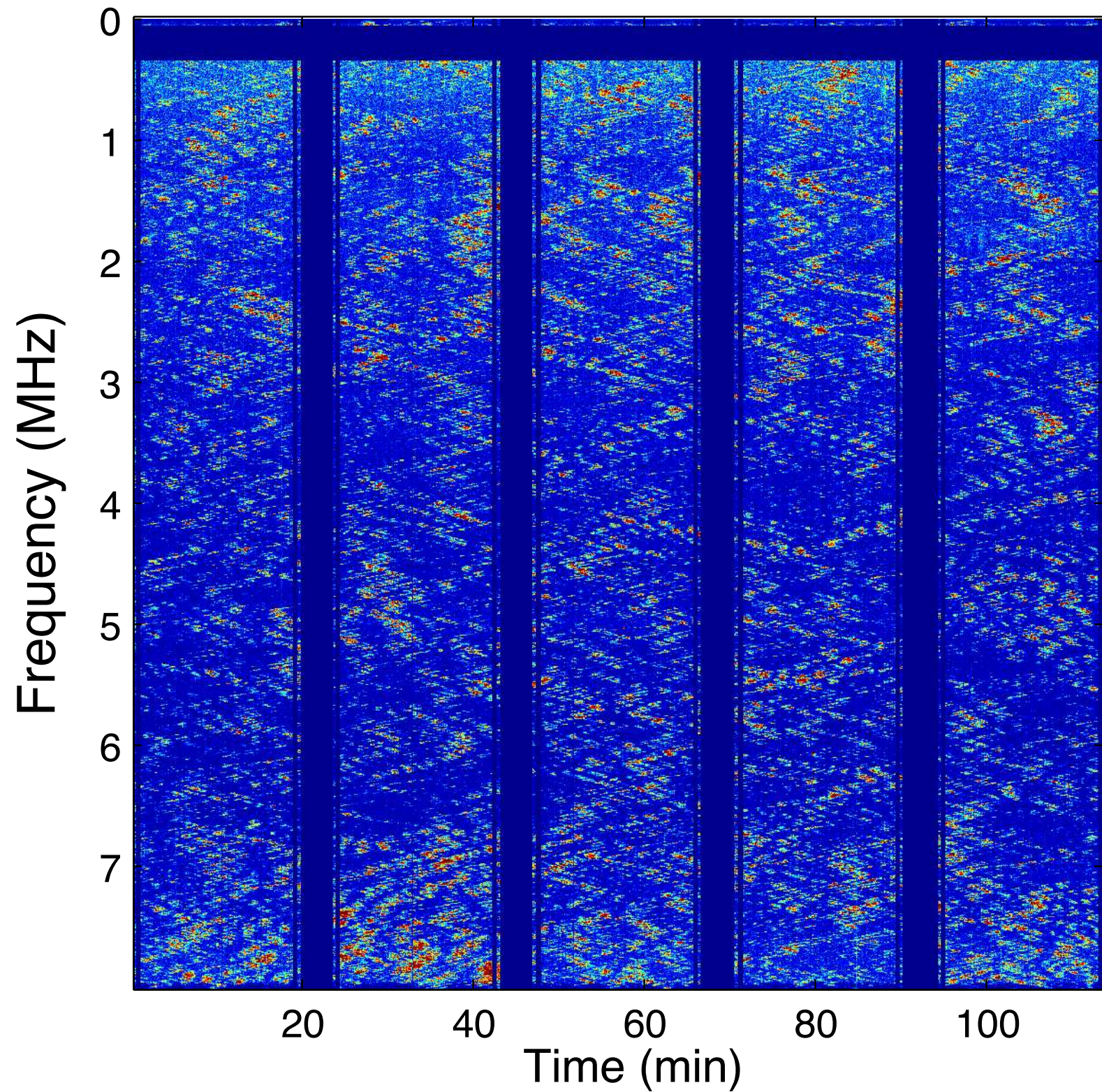


The questions of *homogeneity and isotropy* have come into question as a result of the new analysis technique discovered by Dan Stinebring called the *secondary spectrum* which often displays *parabolic arcs*

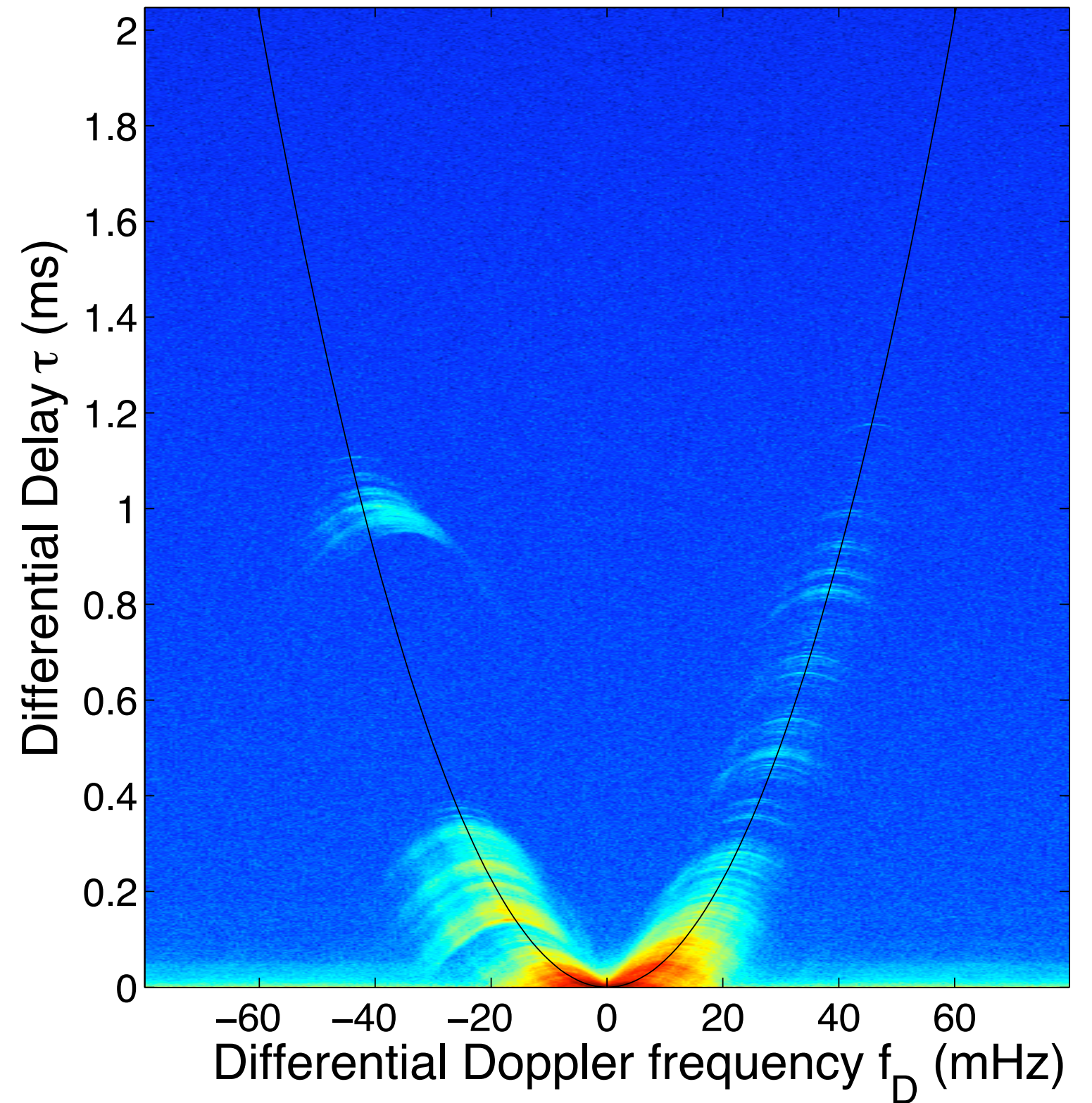
This technique requires a high signal to noise ratio and best results have been obtained at Arecibo. FAST would be ideal.

Parabolic arcs, which are common, can only be observed when the turbulence is highly anisotropic, but methods for estimating the degree of anisotropy are still under development. The problem is complicated by increasing evidence that strong inhomogeneity is also common.

# Dynamic Spectrum of B0834+06 at 327 MHz



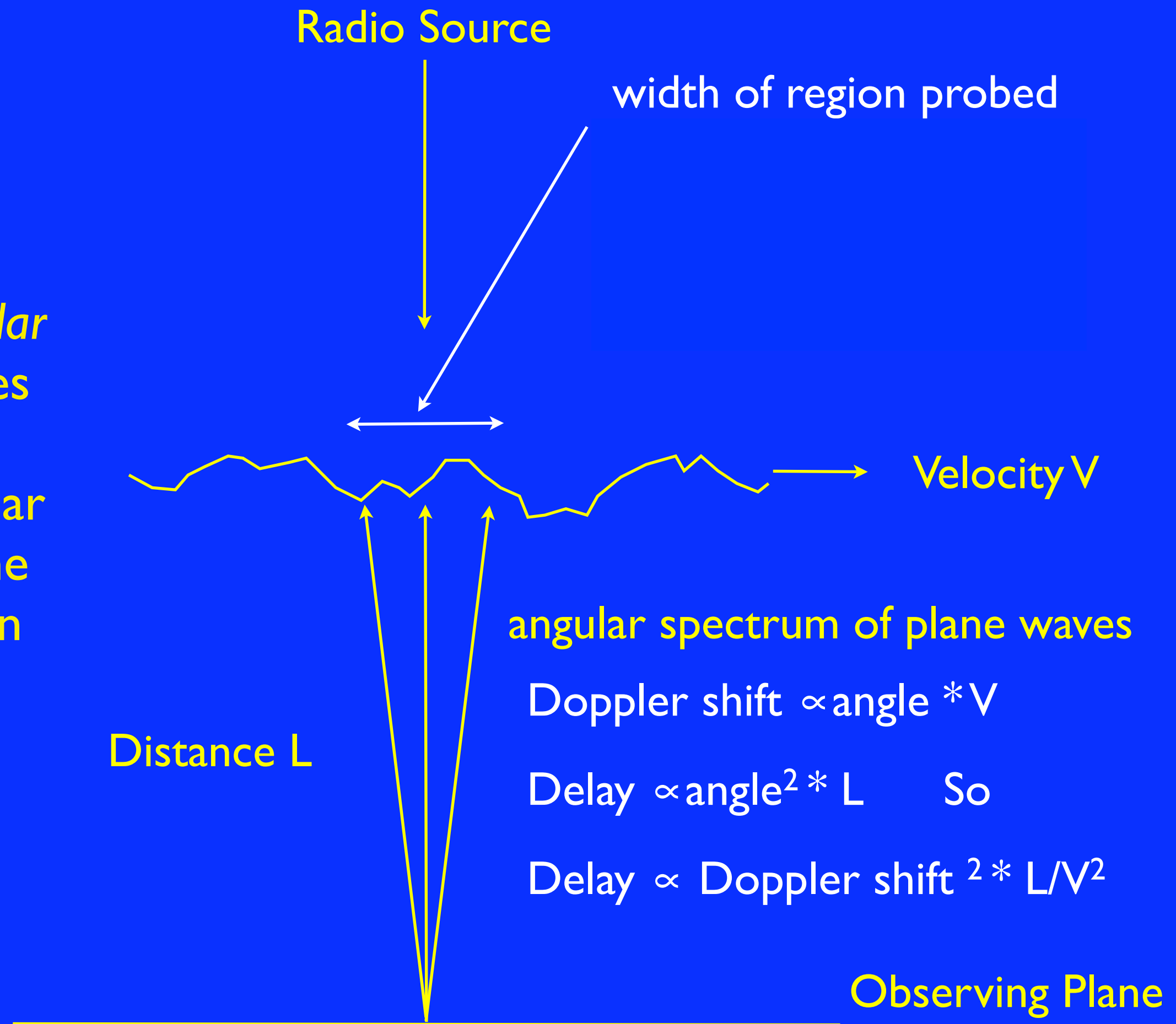
# Secondary Spectrum



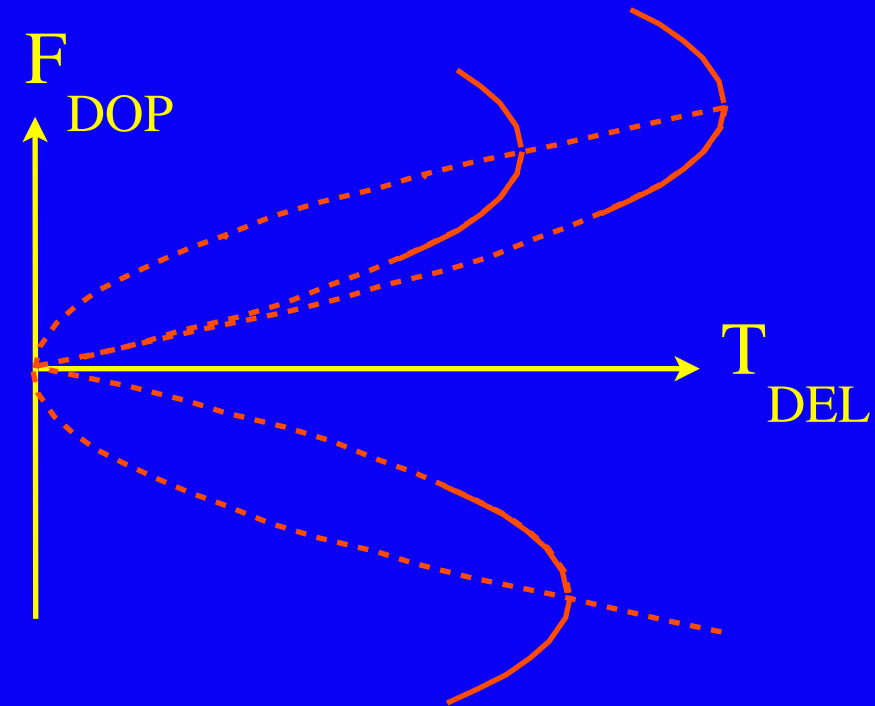
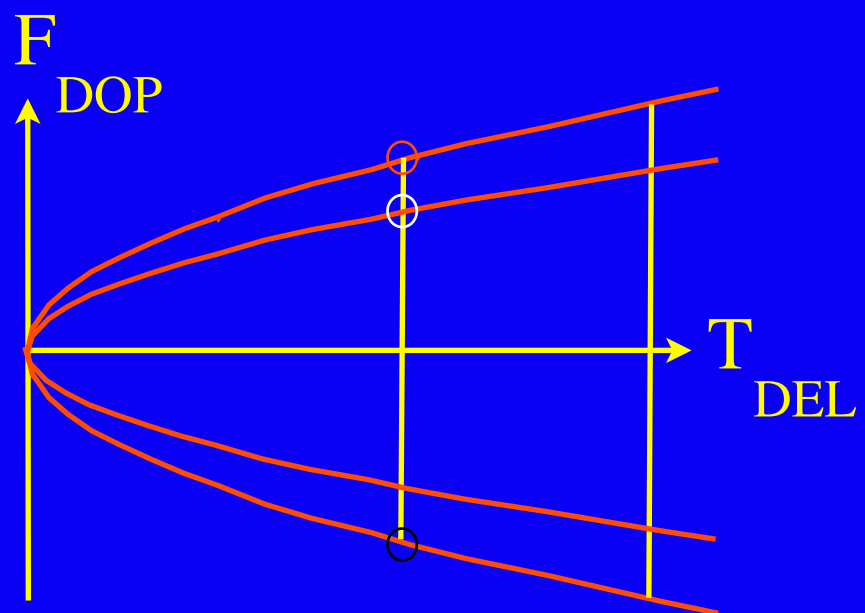
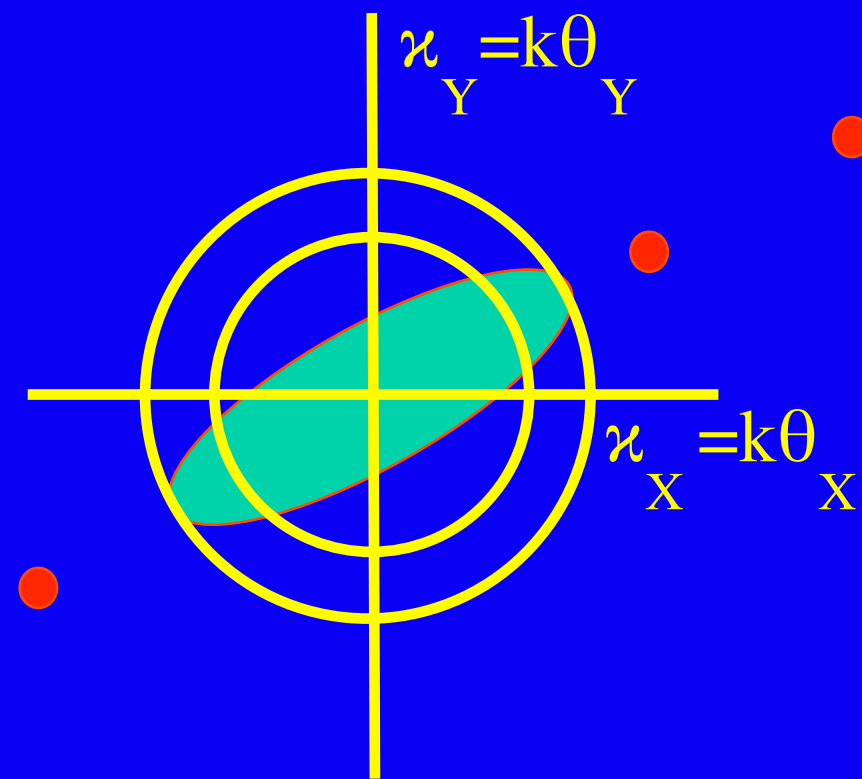
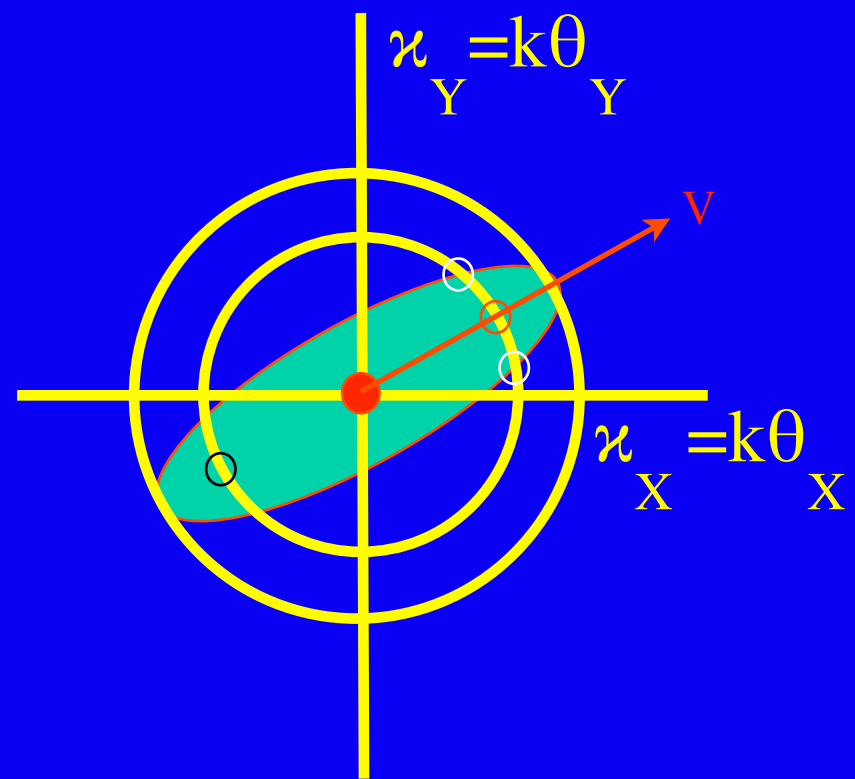
The parabolic arc in the secondary spectrum is the **eigenfunction** of forward scattering

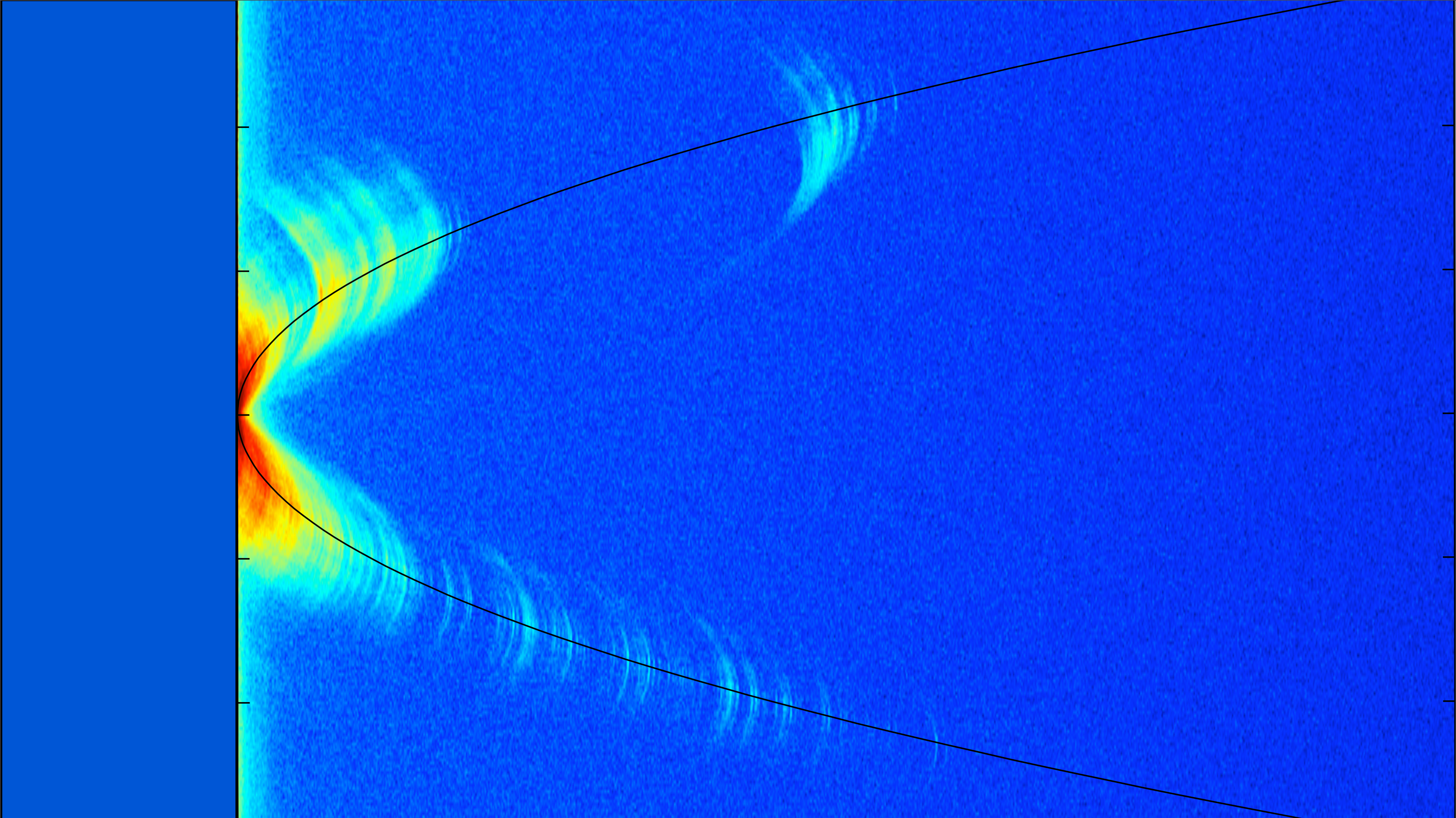
An incident wave is scattered into an *angular spectrum* of plane waves

Each wave in the angular spectrum is seen by the observer with a certain *delay* and *Doppler shift*





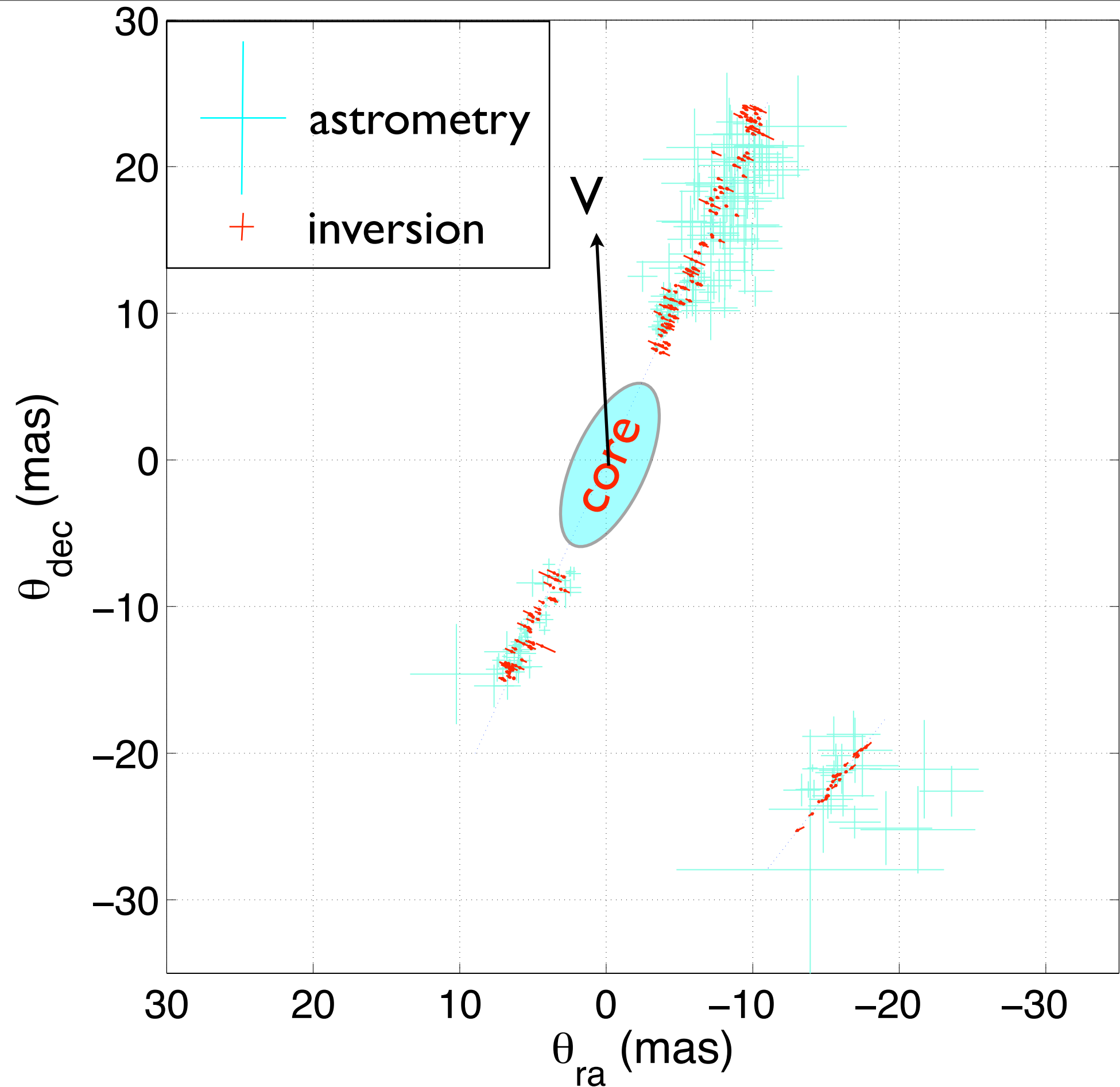




Monday, May 9, 2011

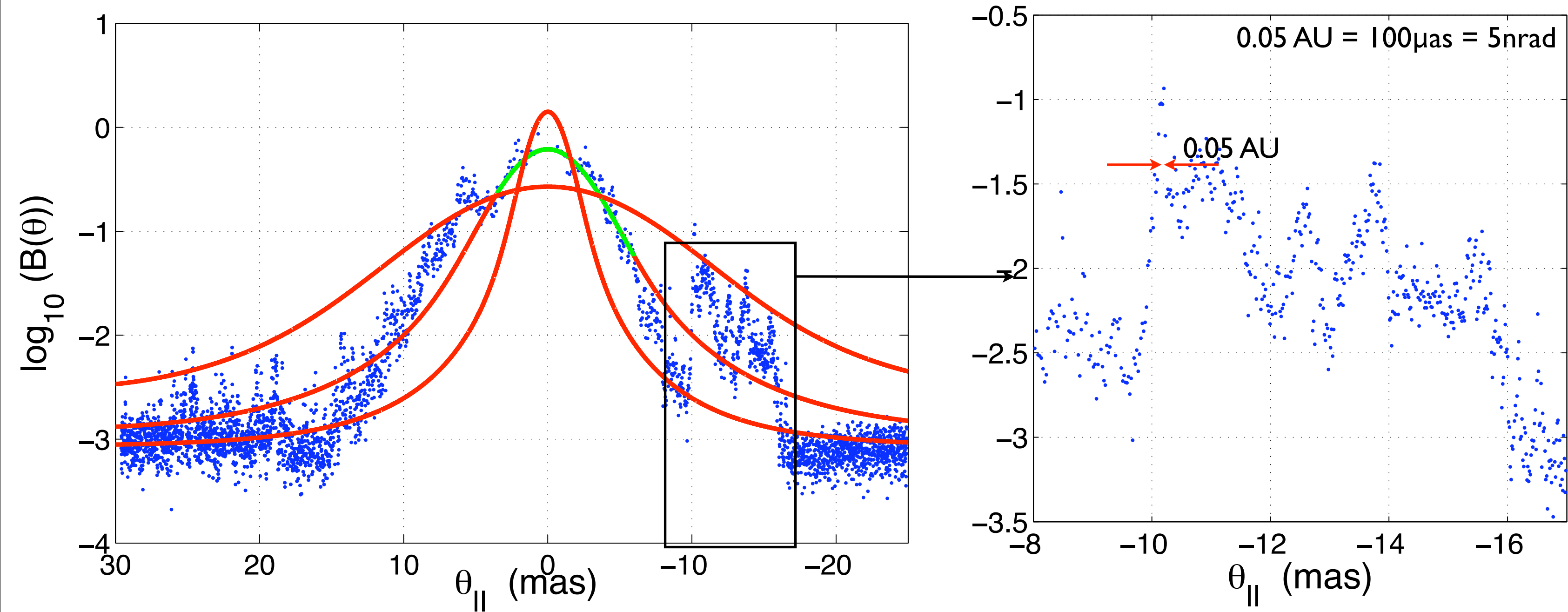
# Reconstructed Image

This type of image reconstruction is possible with a single telescope when the scattering is highly anisotropic, provided the S/N ratio is high.



# Reconstructed one-dimensional brightness distribution of B0834+06

the width in the perpendicular direction is about 1.3 mas



clearly the scattering is highly anisotropic and also highly inhomogeneous  
i.e. the IISM contains much more fine structure than had been thought

How can a structure as small as 0.05 AU contribute as much scattering as the entire 600 pc line of sight?

We model the turbulence on the path  $L \gg L_0$  as a set of  $N$  independent clouds of size  $L_0$ . Each cloud is fully developed turbulence so  $\text{Variance}(n_e) = \langle n_e \rangle^2$ . Thus for each cloud

$$D_{\text{DMC}}(\tau) = 2(n_e L_0)^2 (V \tau / L_0)^{5/3} \text{ for } V \tau < L_0$$

for the entire path the cloud variances add giving

$$\begin{aligned} D_{\text{DM}}(\tau) &= 2(n_e L_0)^2 (L/L_0) (V \tau / L_0)^{5/3} \\ &= 2\langle \text{DM} \rangle^2 (L_0/L) (V \tau / L_0)^{5/3} \end{aligned}$$

This model predicts  $D_{\text{DM}}(\tau=1000\text{d}) = 5 \times 10^{-6}$  for J1939+2134, just as observed.

So we understand why the DM fluctuations observed are so much smaller than  $\langle \text{DM} \rangle$ , it is partly because of incoherent averaging of 1 pc clouds along the line of sight, and partly because  $V T_{\text{OBS}} \ll L_0$

However a tiny *blob* of size  $L_B \ll L_0$  would have

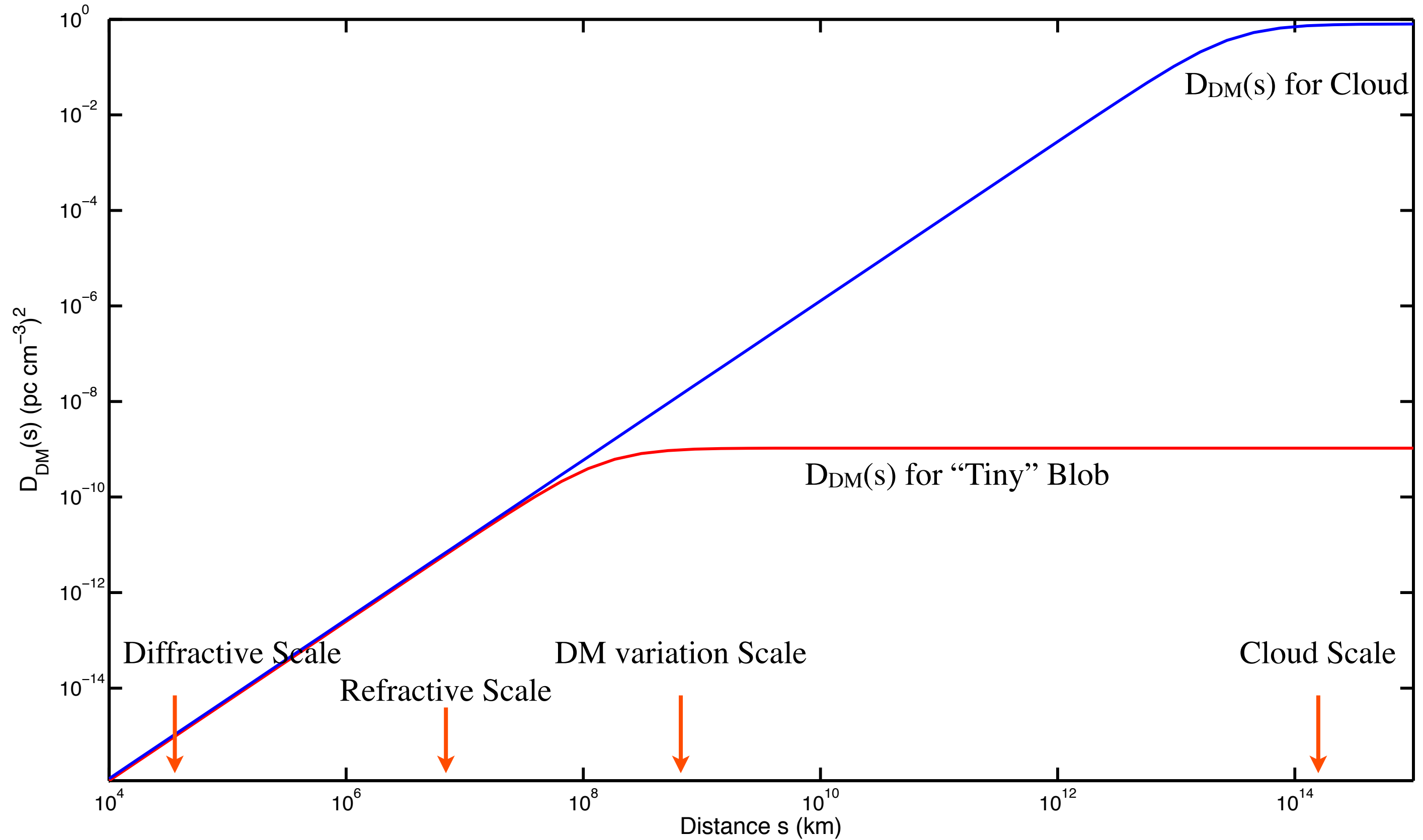
$$D_{\text{DMB}}(\tau) = 2(n_{eB} L_B)^2 (V \tau / L_B)^{5/3}$$

For  $V \tau < L_B$  we have  $D_{\text{DMB}}(\tau) = D_{\text{DM}}(\tau)$  if

$$(n_{eB} / n_e) = L^{1/2} / L_0^{1/3} L_B^{1/6}$$

So the 0.05 AU blob seen in B0834+06 requires only  $n_{eB} \approx 6.8$  !

# Structure Function $D_{\text{DM}}(s)$ for $\text{DM}(t)$



Structures such as this 0.05 AU blob are common in Stinebring's observations. They may be related to *intermittency* in hydrodynamic turbulence.

These blobs could be vortices generated by a shear instability. Such instabilities start at small scales and gradually increase in size and amplitude until the driving shear goes away. They can then persist for a very long time.

Even when the velocity dies away the density fluctuation can persist as “fossil turbulence” confined by the magnetic field.



# Summary:

There are many features of the IISM now measurable. The PTAs, existing and those being developed, will greatly improve our understanding of the IISM and astrophysical plasmas in general.

They may even detect gravitational waves. It is a very exciting time to be studying pulsar timing!