Lectures on radio astronomy: 2

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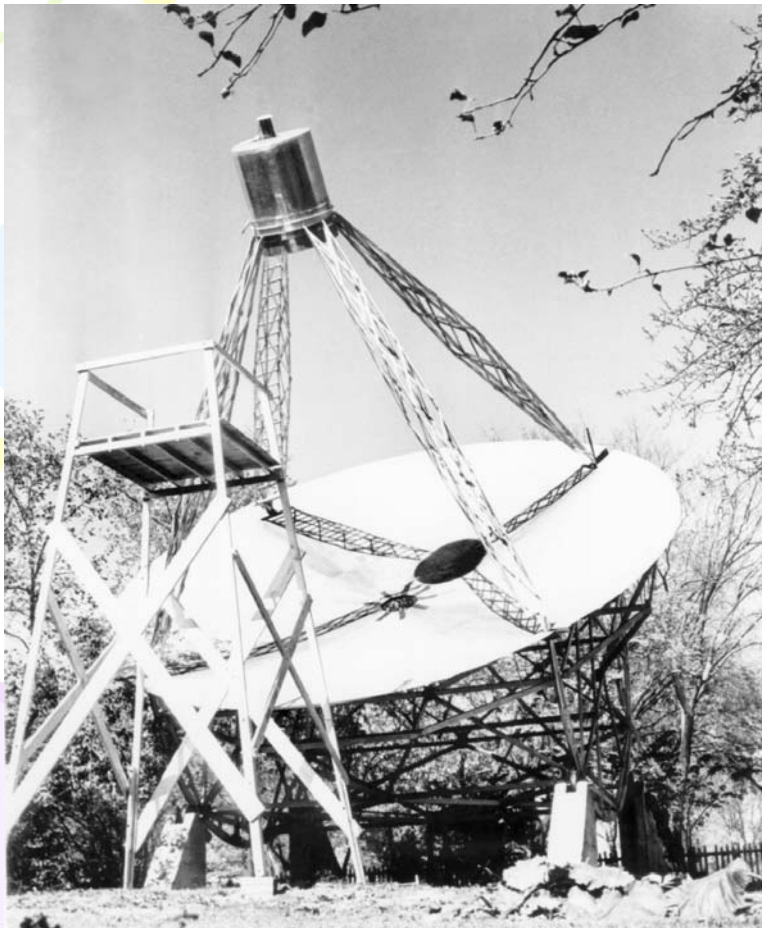
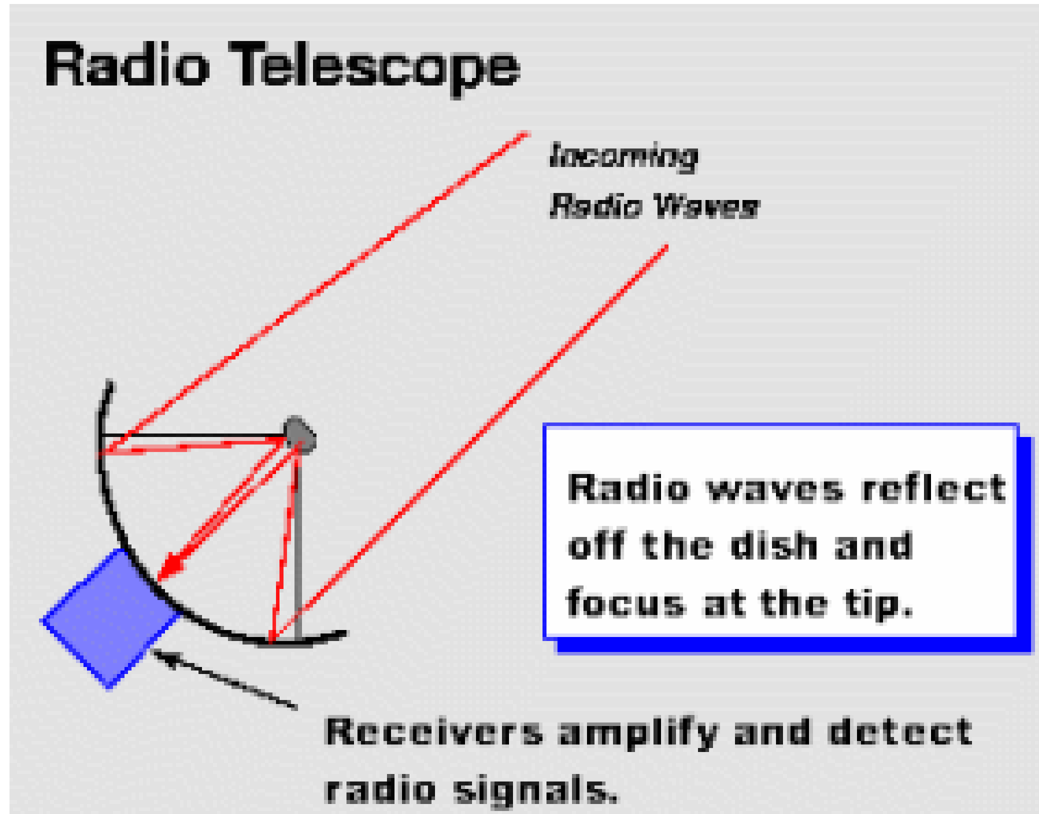
Single element telescopes

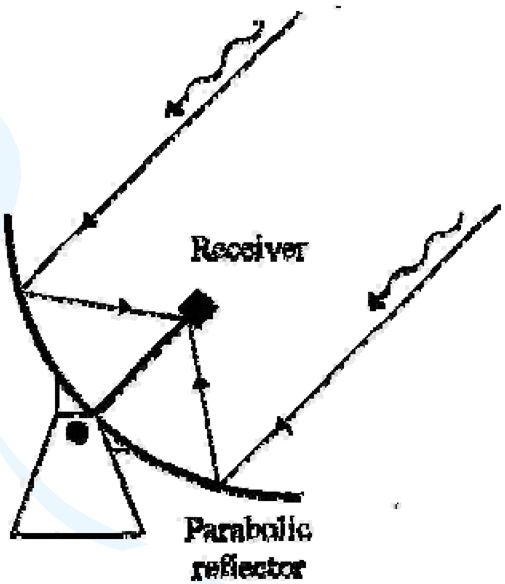
How a parabolic reflector works is just geometry

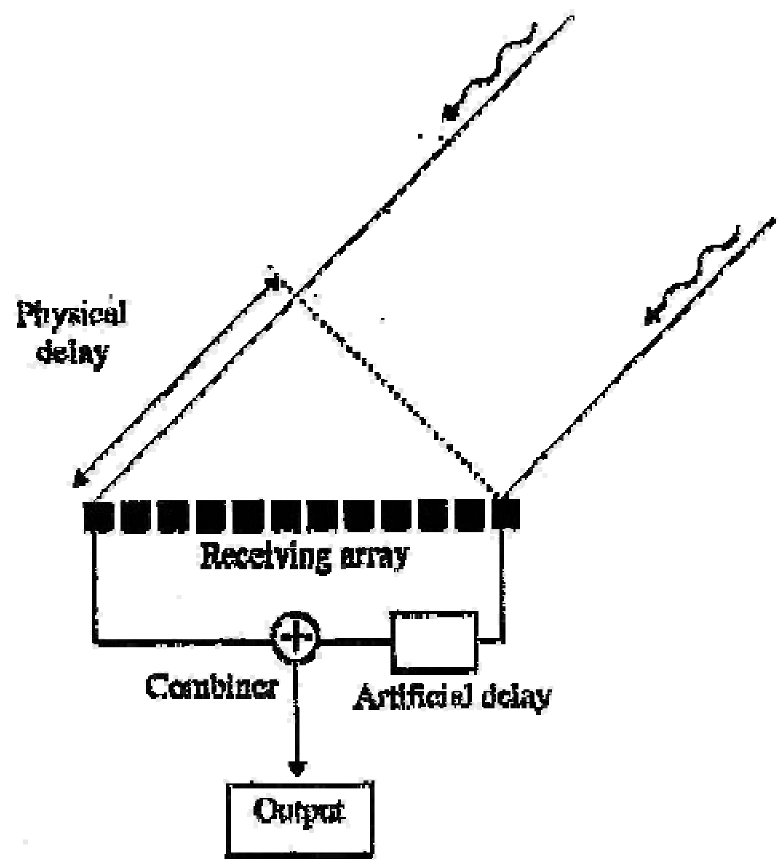




We need to understand how all antennas work

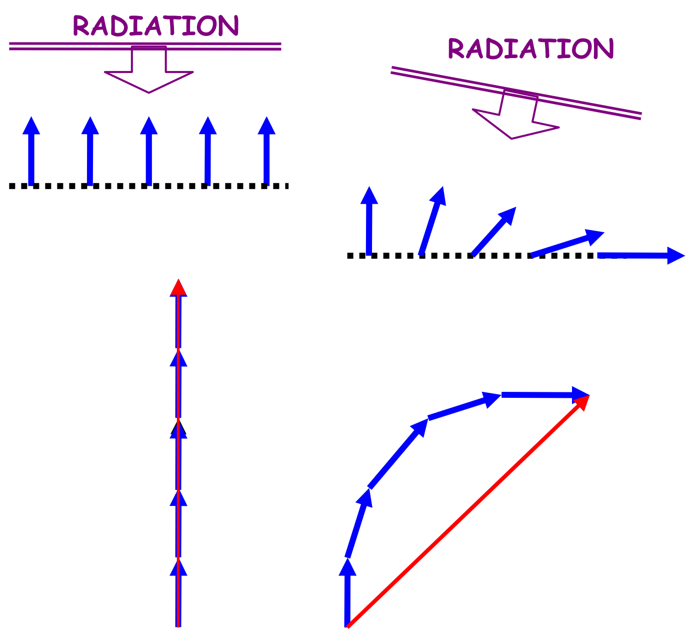
Imagine the antenna split up into several segments





This is what happens to beam response as we go off axis

On axis – beam center
All in phase:
Maximum signal



Off axis – beam edge
Out of phase:
Zero signal

The length of the vector as function of angle is sin θ/θ
This is the Fourier transformation of the "top hat" – ∏

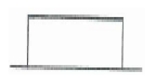
The response of an antenna

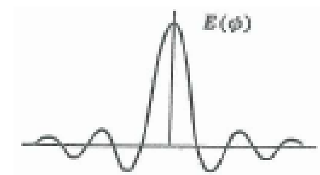
- Determined by the electric field distribution over the aperture, $E(x)$
- The beam is the Fourier transform [FT] of $E(x)$: $b(\theta) = \int E(x)\, e^{2\pi i x \theta}\, dx$
  or, $E(x) \rightarrow b(\theta)$ [$\rightarrow$ = FT]
- $b(\theta)$ is the voltage beam
  The power beam – $b^2(\theta)$ – is found from the FT of the autocorrelation:
  $\int E(l)\, E(l+x)\, dl = E(x) \star E(x)$

Aperture illumination and beam related through FT

$E(x) \rightarrow b(\theta)$

Voltage

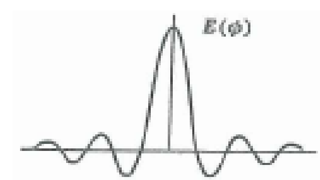




$\sin(\theta)/\theta$

$\star$ $\times$

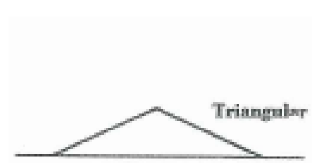
Voltage

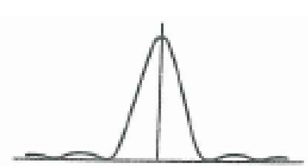


$\mathrm{sinc}(\theta)$

= =

Power – $b^2(\theta)$

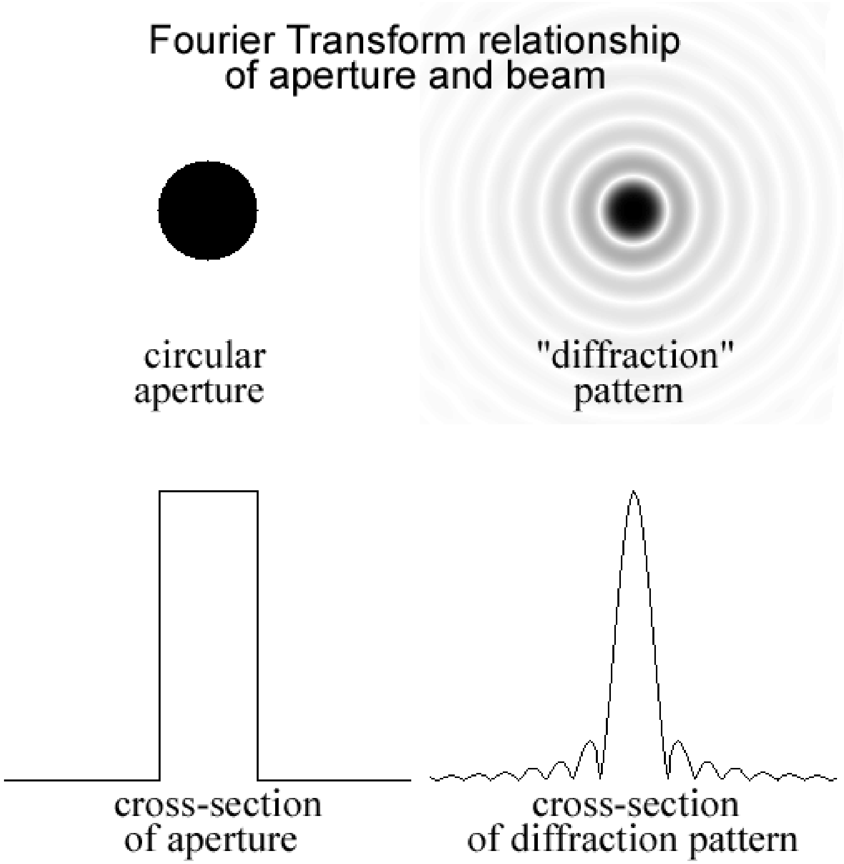




$\mathrm{sinc}^2(\theta)$

So an antenna Fourier transforms the illumination

- When the vectors curl up to 0, one edge is 360^{o} out of phase with other – this is first null.
- When vectors curl up twice, 2nd null
- See that beam size depends on D/λ



Illumination usually not uniform – can vary it, too

- (sin θ)/θ is the voltage beam
- Power beam is ($\sin^2\theta$)/θ^2
- Most feed systems taper illumination at edge
- Less spillover, lower sidelobes, but larger beam

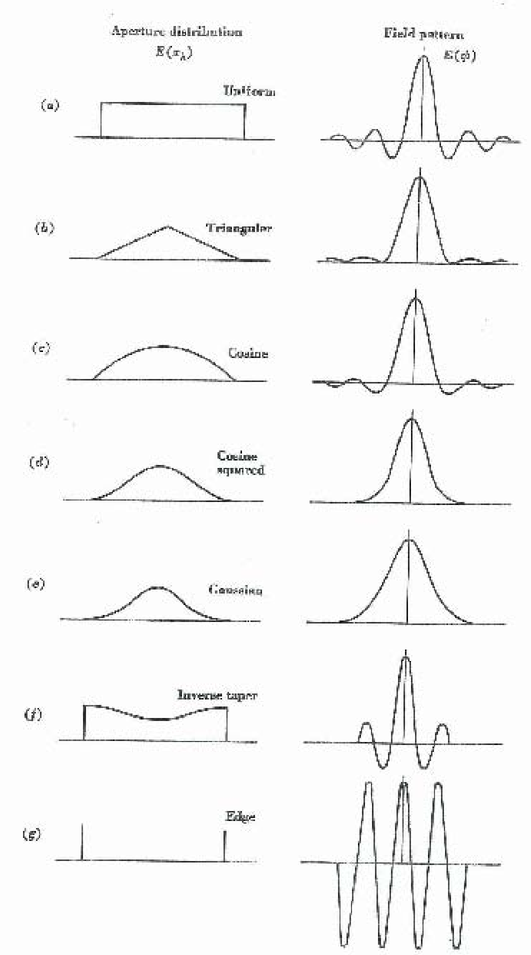
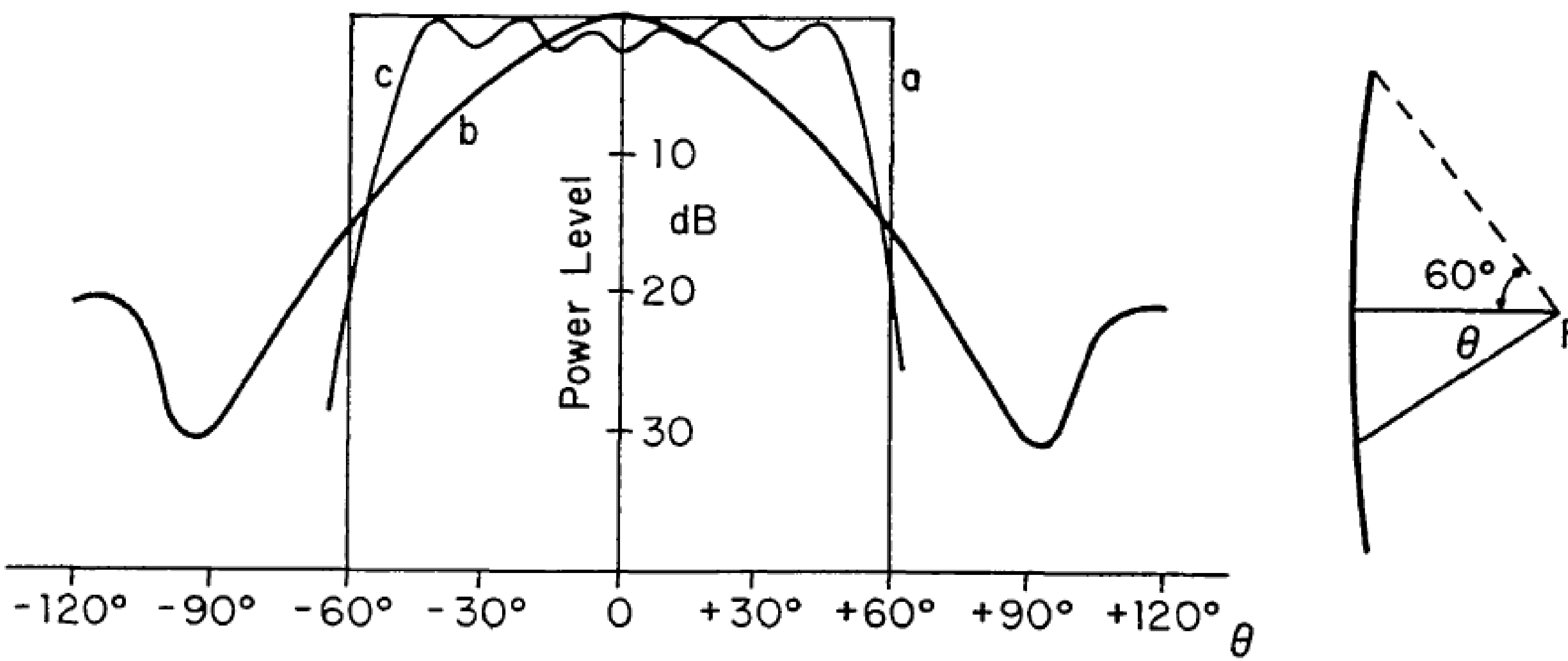
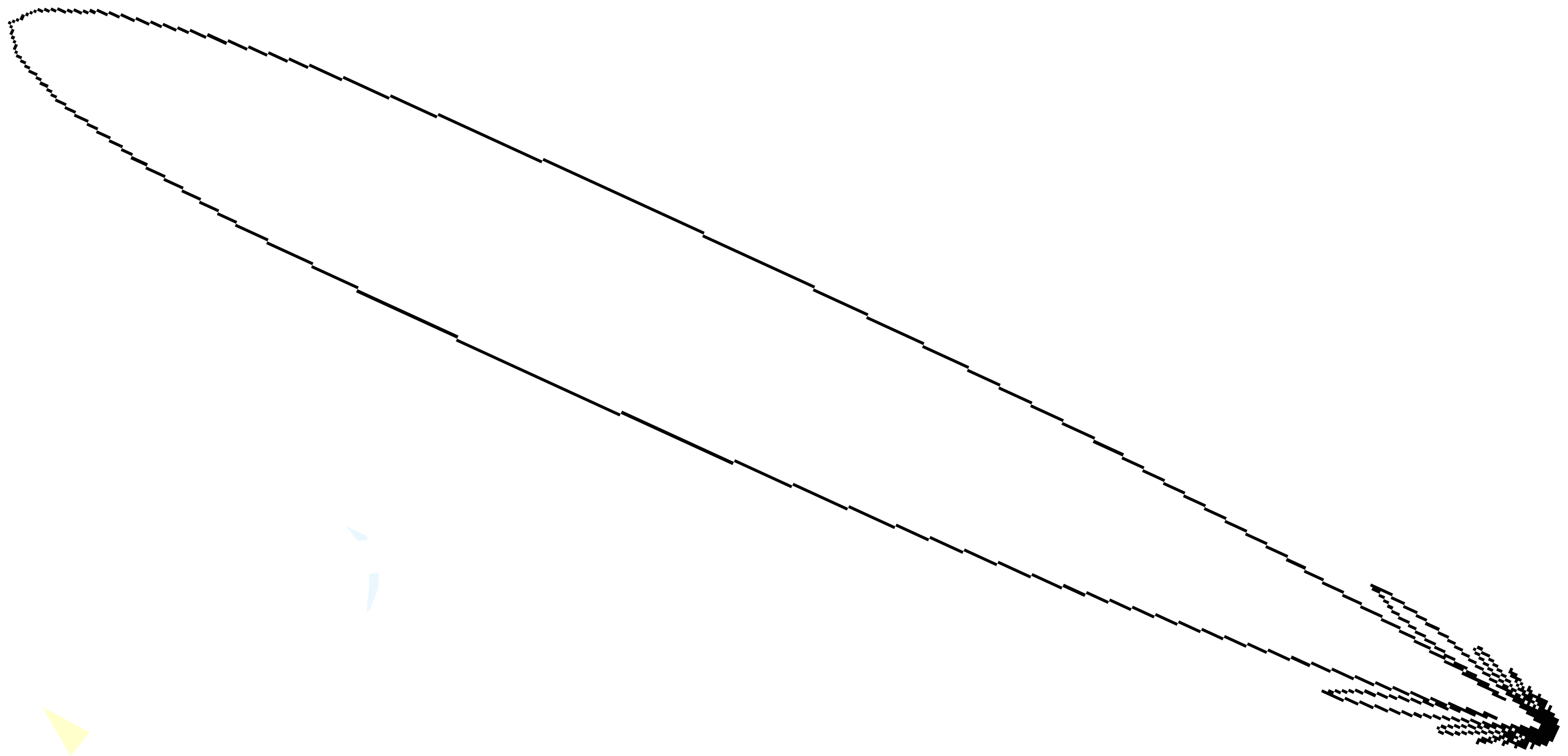


Fig. 6-9. Different aperture distributions with associated antenna patterns

Illumination patterns for a parabolic reflector



Here's a telescope beam in angular coordinates



Observation: convolve the sky emission by the beam

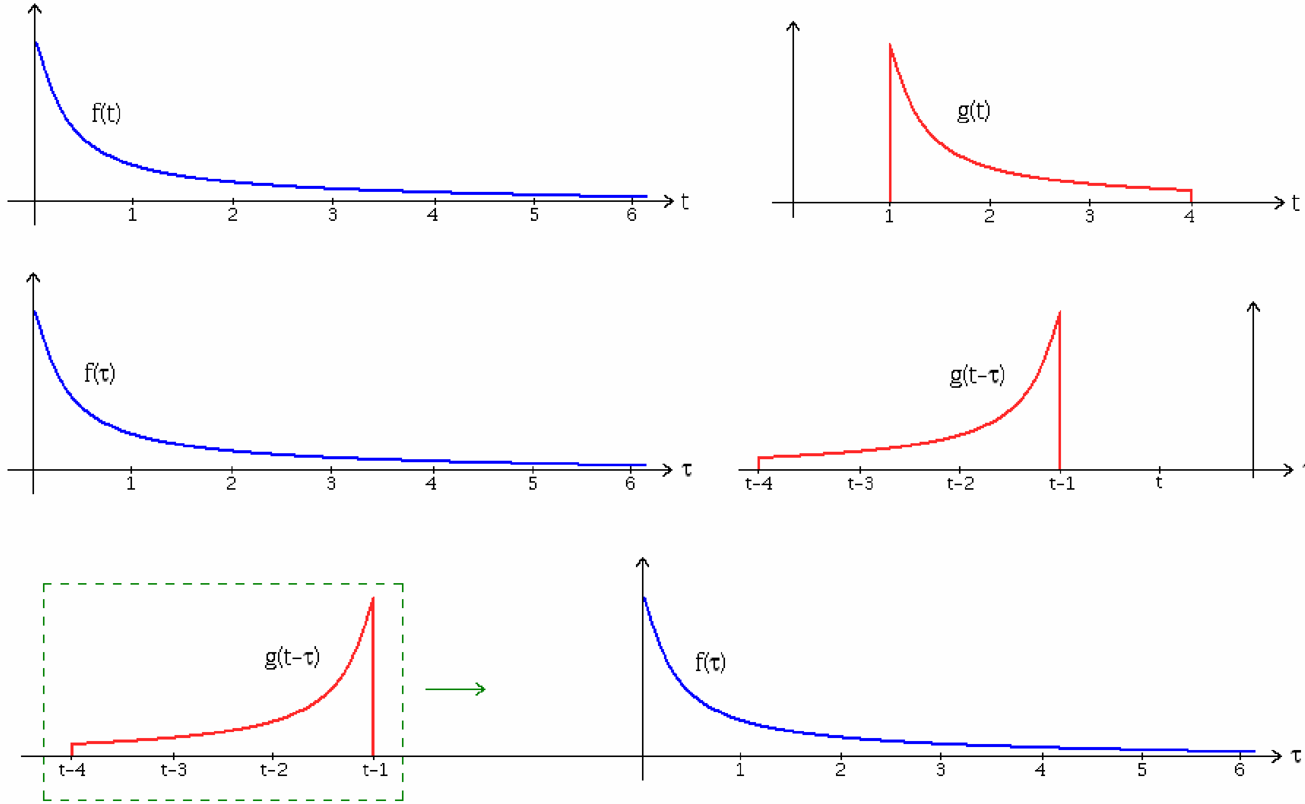
- The power beam – $b^2(\theta)$ – obtained from FT of autocorrelation of $E(x)$: $\int E(l)\, E(l+x)\, \mathrm{d}l = E(x) \star E(x)$
- What an antenna actually "measures" is the convolution of the sky intensity distribution – $I(\theta)$ – with the beam pattern: $B(\theta) * I(\theta) = \int B(\varphi)\, I(\varphi-\theta)\, \mathrm{d}\varphi$
- The difference between convolution and correlation is the reversal of one function

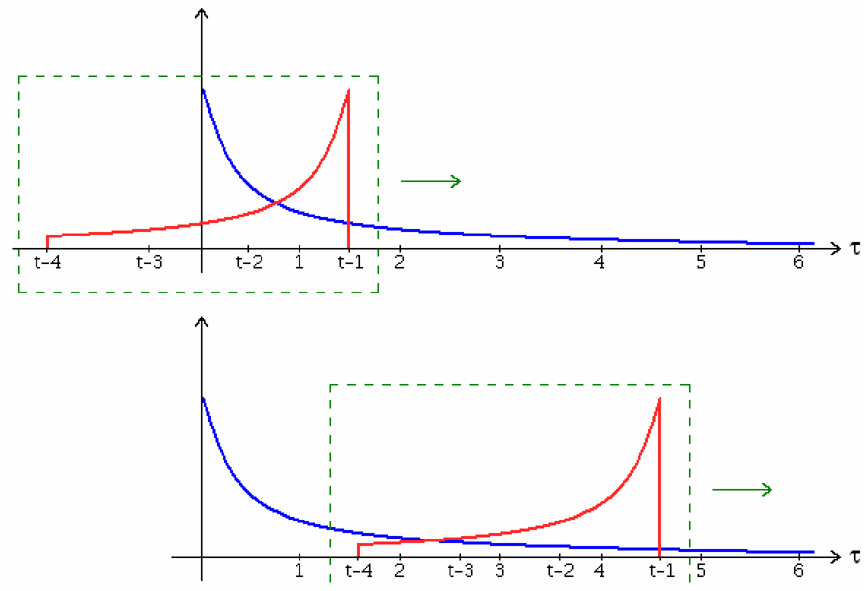
Let's look more closely at convolution

- FT: $g(t) = \int G(f)\, e^{2\pi i f t}\, df : G(f) \rightarrow g(t)$
- Convolution:

$$g(t) * h(t) = \int g(x)\, h(t-x)\, dx$$
$$= \int g(x)\, [\int H(f)\, e^{2\pi i f (t-x)}\, df]\, dx$$
$$= \int [\int g(x)\, e^{-2\pi i f x}\, dx]\, H(f)\, e^{2\pi i f t}\, df$$
$$= \int [G(f)\, H(f)]\, e^{2\pi i f t}\, df$$

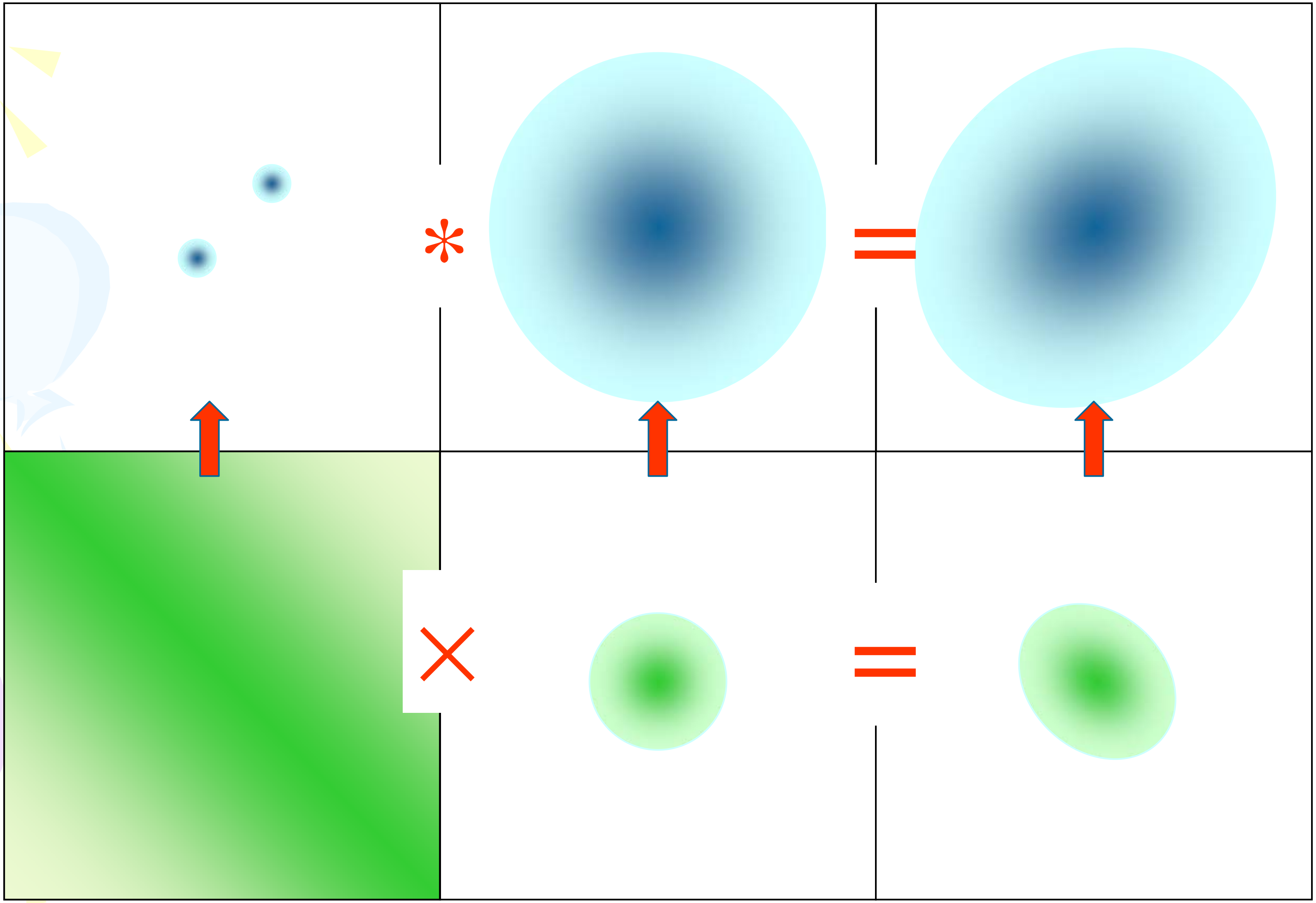
- so, $g(t) * h(t) \leftarrow G(f) \cdot H(f)$
- Often, take: convolution $\equiv$ correlation





Convolution of one function by another

Illustration of the FT and image convolution relation



Observation: convolution of source by telescope beam

- This can also be seen as taking FT of source brightness (=visibility)...
- ...multiplying it by the FT of the telescope response (or beam)...
- ...and FT the result back to the image plane.
- May seem complicated, but fundamental to interferometers.
- We will return to this.

Derivation of the basic antenna equation for S & T_a

$$\text{Planck}: B = \frac{2h\nu^3}{c^2}\left(e^{-h\nu/kT} - 1\right)^{-1},$$

$$\mathrm{W\ m^{-2}\ Hz^{-1}\ sr^{-1}}$$

$$\text{Rayleigh-Jeans}: \ h\nu \ll kT \ (\text{"radio"})$$

$$B \approx \frac{2h\nu^3}{c^2}\frac{kT}{h\nu} \quad \left[\text{NB}: e^{-h\nu/kT} \approx 1 + \frac{h\nu}{kT}\right]$$

$$= \frac{2\nu^2 kT}{c^2} = \frac{2kT}{\lambda^2} \qquad \left[\frac{\nu}{\mathrm{c}} = \frac{1}{\lambda}\right]$$

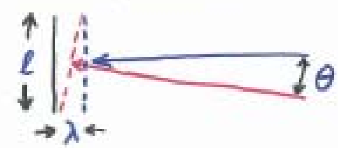
$$\text{Flux density}: S = \int B\, \mathrm{d}\Omega = \frac{2kT\Omega}{\lambda^2}$$

Compact sources

$\Omega_s \leq \Omega_a$

Flux density: $S = \int B_s \, d\Omega = \frac{2kT_s}{\lambda^2}\Omega_s$

Telescope beam:



$\sin\theta = \lambda/\ell$

$\theta \simeq \lambda/\ell$

$(\theta \simeq \ell_\lambda^{-1})$

2-D: $\theta^2 \simeq \lambda^2/\ell^2 \Rightarrow \Omega_a \simeq \lambda^2/A$

beamwidth

antenna area

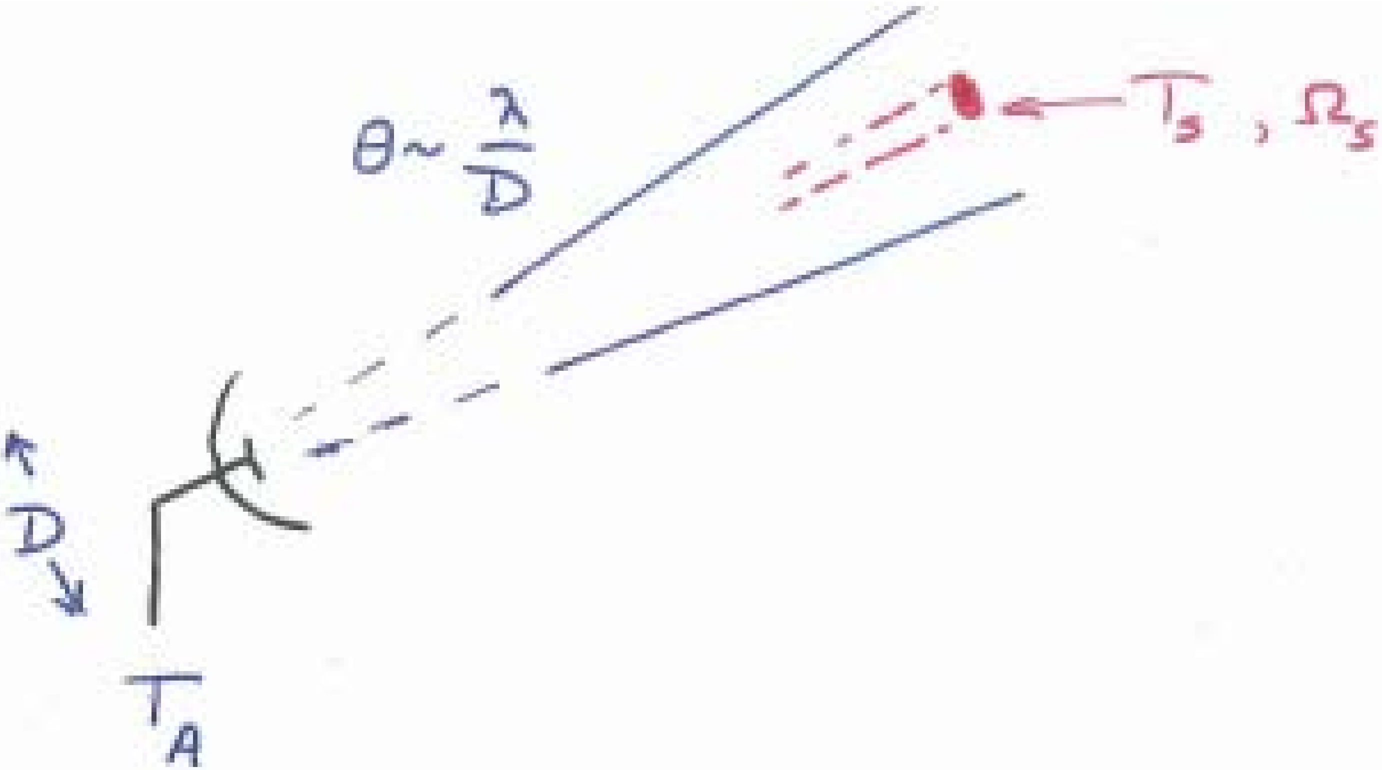
$$\boxed{S = \frac{2kT_a}{\lambda^2}\Omega_a = \frac{2kT_a}{A} \ \mathrm{W\ m^{-2}\ Hz^{-1}}}$$

For many discrete sources: $S \sim 10^{-26}\ \mathrm{W\ m^{-2}\ Hz^{-1}}$

Definition: 1 Jansky (Jy) $= 10^{-26}\ \mathrm{W\ m^{-2}\ Hz^{-1}}$

(previously: flux unit = f.u.)

Justification for replacing T_s and Ω_s by T_a and Ω_a



$$T_s \simeq T_A \frac{\theta^2}{\Omega_s} \qquad (\Omega_s < \theta^2)$$

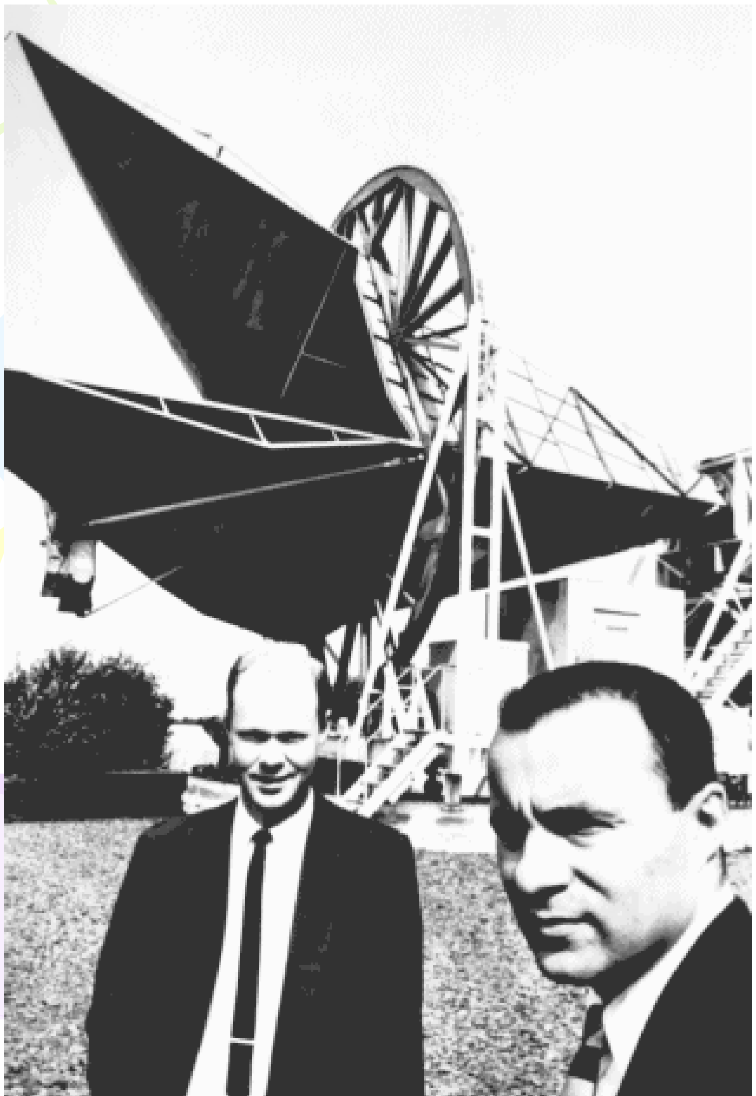
For a broad, uniform source, antenna size doesn't matter

Since $T_a = \dfrac{T_s \Omega_s}{\Omega_a}$, for $\Omega_s \geq \Omega_a$,

$T_a = T_s$ (in a perfect antenna).

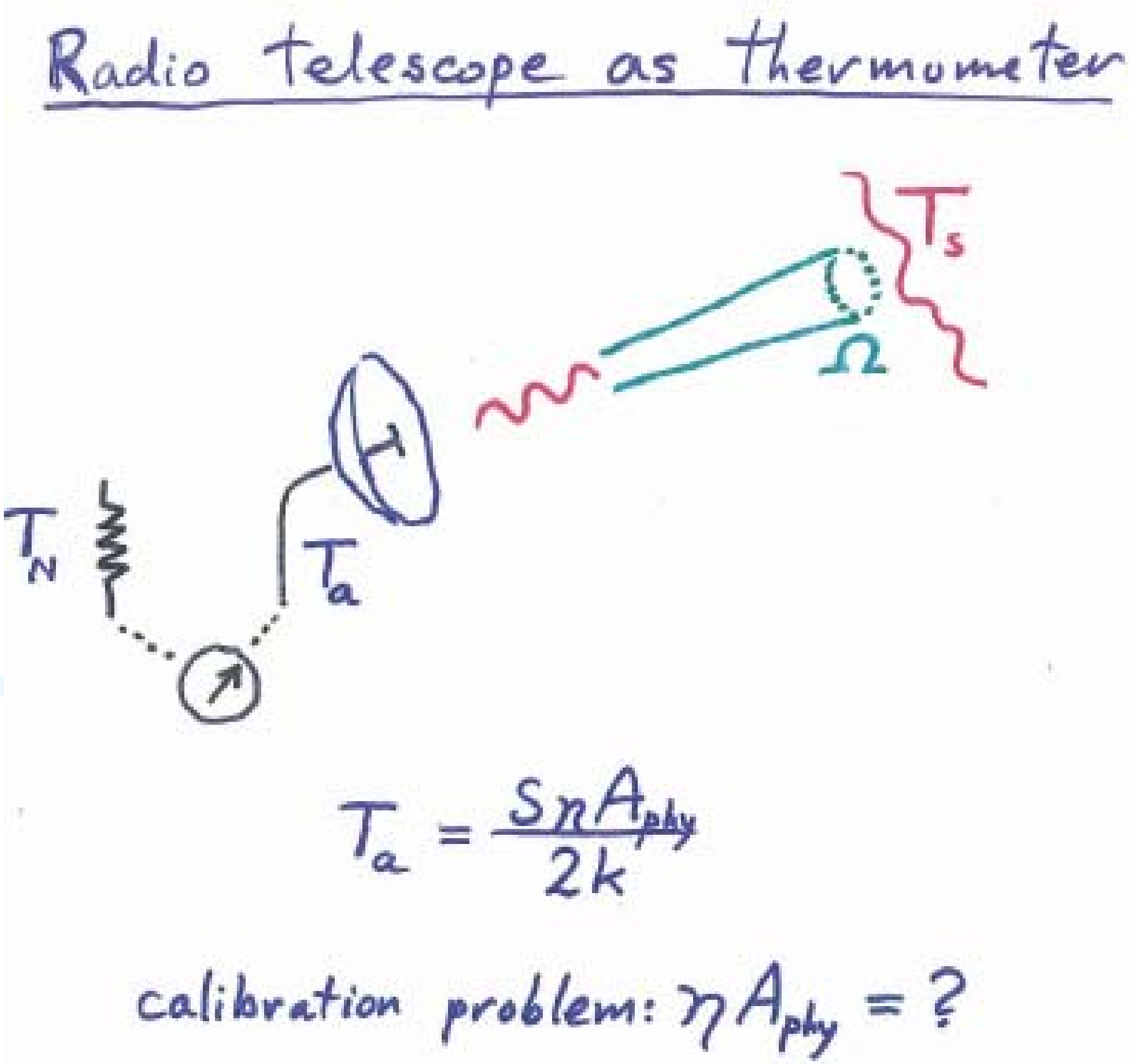
Moreover, note that $A\Omega_a$ $(= \lambda^2)$ is constant, so increasing antenna area (A) will not increase signal power, P $(= kT_a)$.

So for CMB detection, large & small horn gave same signal



R. H. Dicke and his colleagues calibrating a microwave radiometer using an ambient temperature absorber (Dicke is holding this panel, then referred to as a 'shaggy dog'. The photo dates from the mid-1940s. At about this same time (1946) Dicke *et al.* established an upper limit of 20 K on the cosmic background at microwave frequencies using similar apparatus.

Our telescope measures the sky temperature



Effective area and the system equivalent flux density (SEFD)

$$S = \frac{2kT_a}{A} \quad (\mathrm{W\,m^{-2}\,Hz^{-1}}) = 10^{26}\,\mathrm{Jy}$$

$$k = 1.38\times10^{-23}\ \mathrm{J\,K^{-1}}$$

$A \rightarrow$ area $= 490\ \mathrm{m^2}$ (25 m dish)

$$A_{phy} = 490\ \mathrm{m^2} \Rightarrow A_{eff} < A_{phy}$$

← 25 m → ← ~16 m →

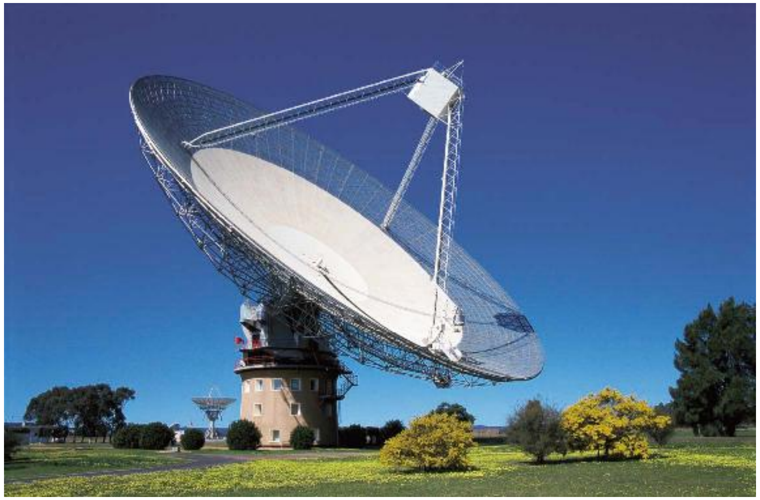
$$A_{eff} = \eta A_{phys} \qquad 0.4 \lesssim \eta \lesssim 0.7$$

$$\text{25 m dish: } \frac{S}{T_a} = \frac{2\times1.38\times10^{-23}}{245\ \mathrm{m^2}} = 11\times10^{-26} \simeq \frac{10\ \mathrm{Jy}}{\mathrm{K}}$$

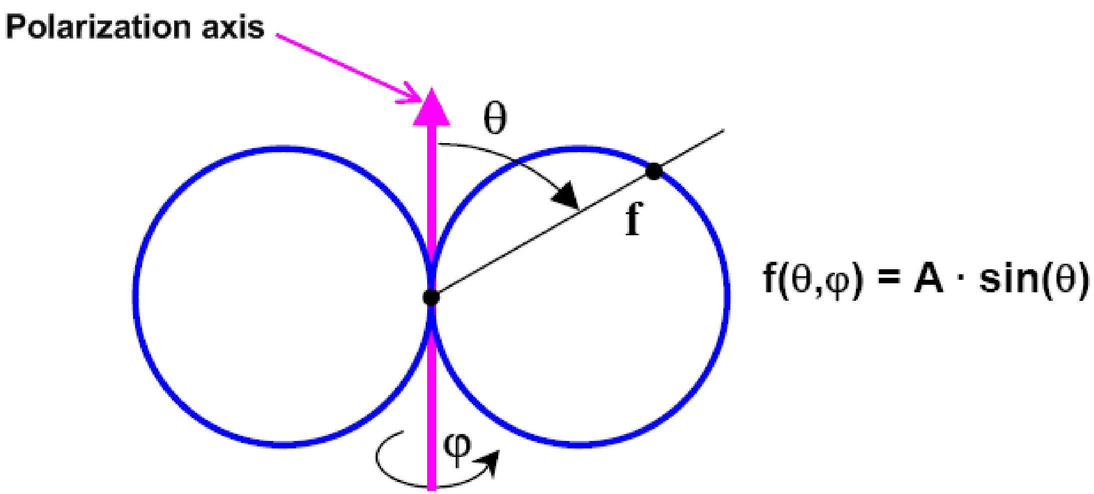
Size	S/T (Jy/K)	Telescope
25 m	10	Dwingeloo
94	0.8	Westerbork
100	0.7	Effelsberg
200	0.18	Arecibo

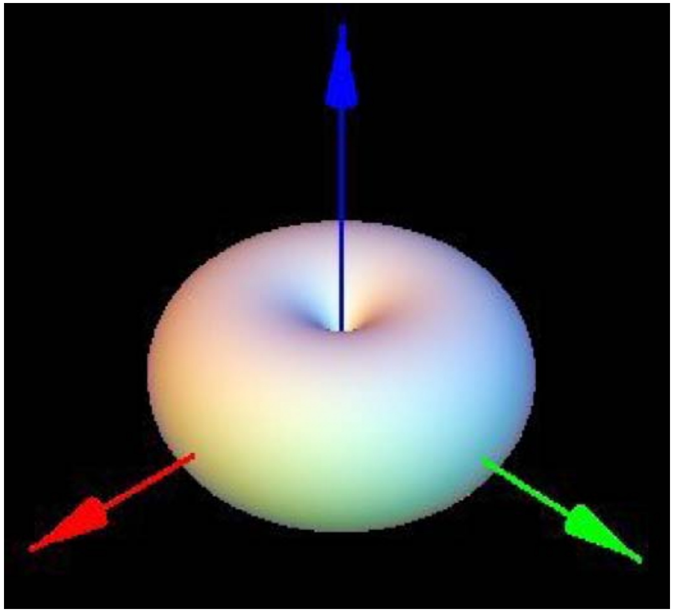
Collecting area? Might guess something like physical area

- For a parabola, the effective area (A_e) is always less than the physical area
- For a dipole, the effective area is roughly, $A_e \sim \lambda^2$
- Dipoles are most effective at long wavelengths!



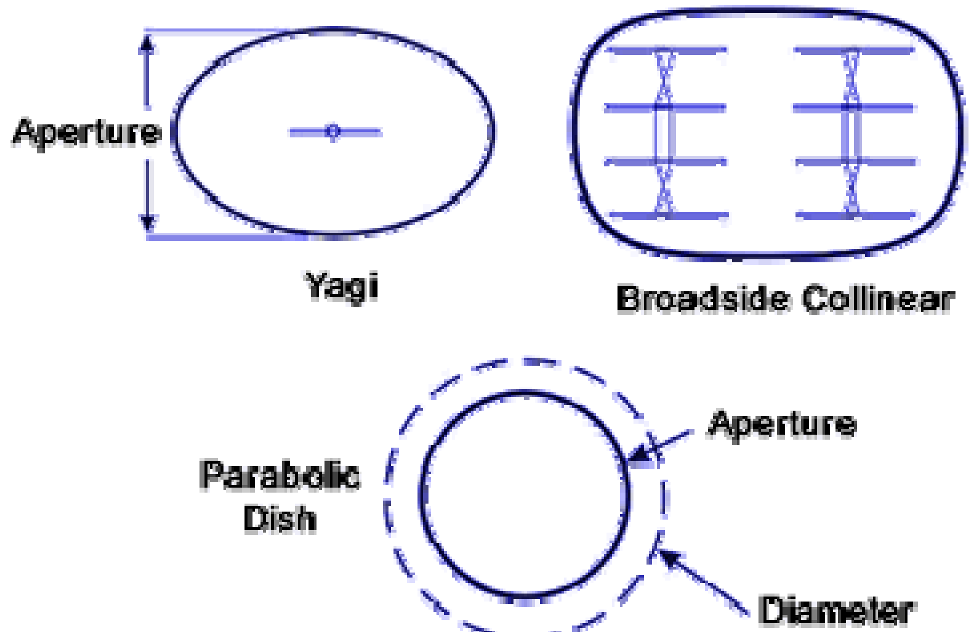
Effective area of dipole



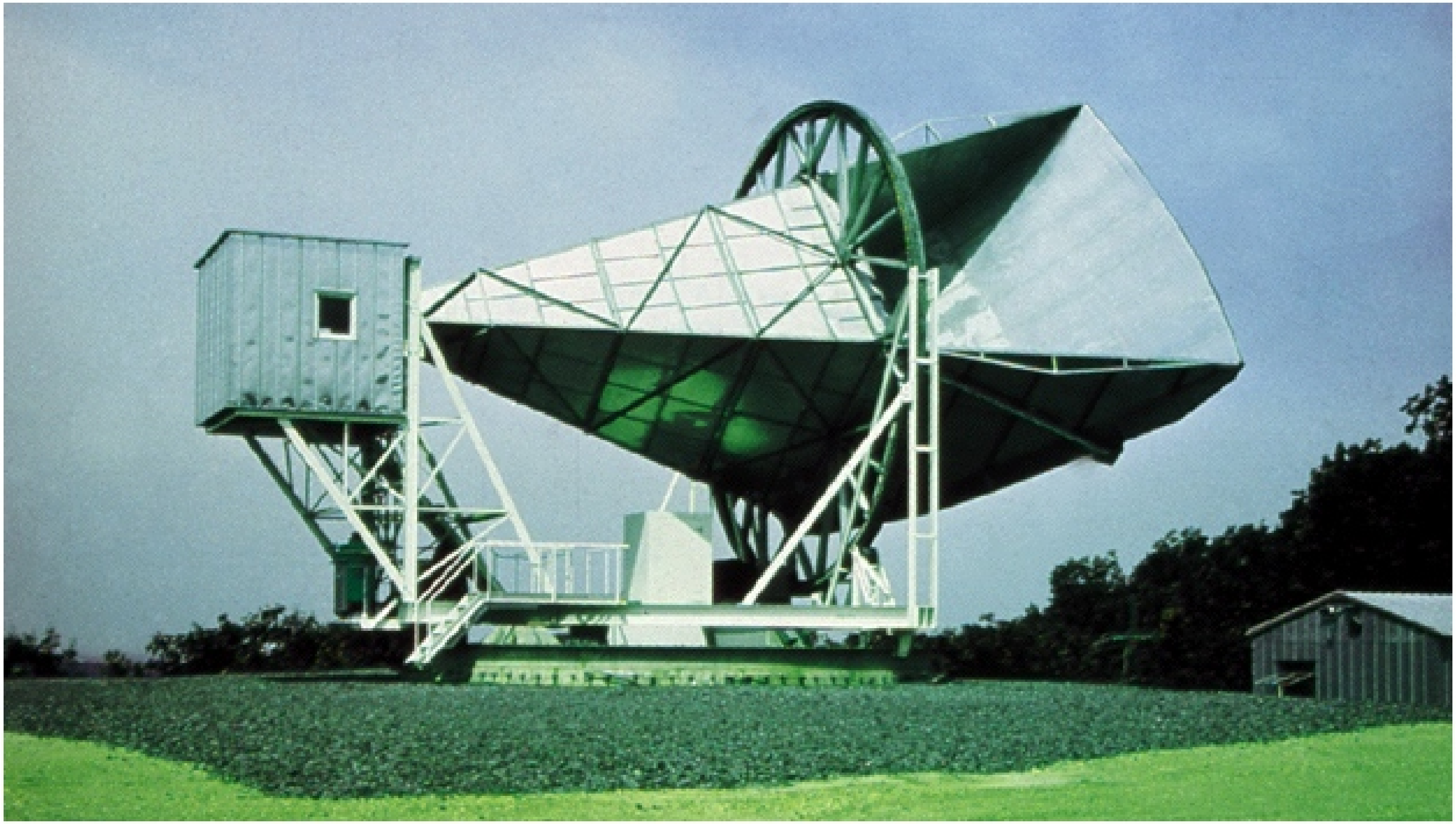


- For any antenna, beam size $\theta \approx \lambda/d$; $\theta_1\theta_2=\Omega$, so $\Omega \approx \lambda^2/d_1d_2$; effective area $A_e \approx d_1d_2$
- Dipoles of any size have same beam, Ω
- So, Ω is constant, and we have $A_e \approx \lambda^2/\Omega$
- The result is that A_e increases with λ^2

Effective aperture (area) for different antenna types

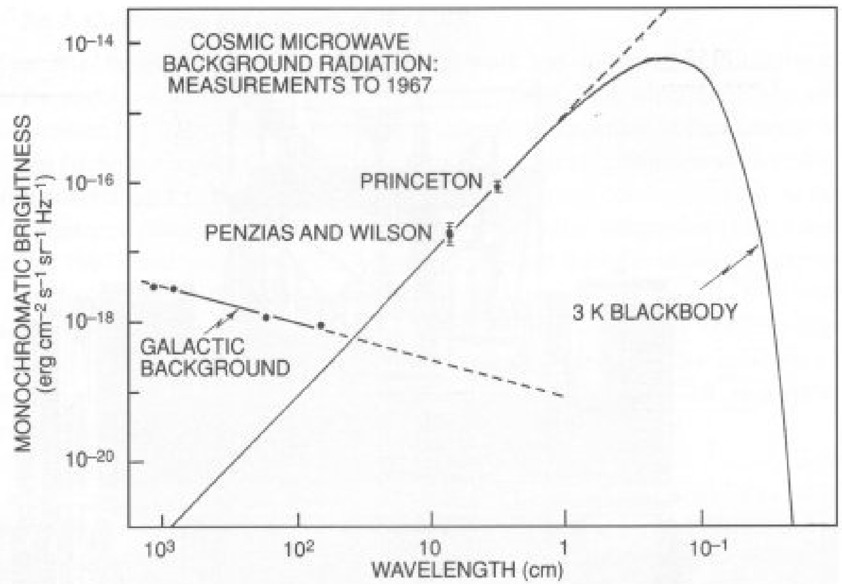


In fact, only horns have $A_e \approx$ physical area



Absolute flux density determinations are difficult

- This is why CMB measurement didn't happen sooner
- Horns usually used at high frequencies
- Dipoles are usually used at the lower frequencies



A second measurement of the CBR at 3.0 cm (Roll and Wilkinson, 1966) confirms the discovery of a thermal background and refines the value for T_0.

From the SEFD and system noise, derive observing time

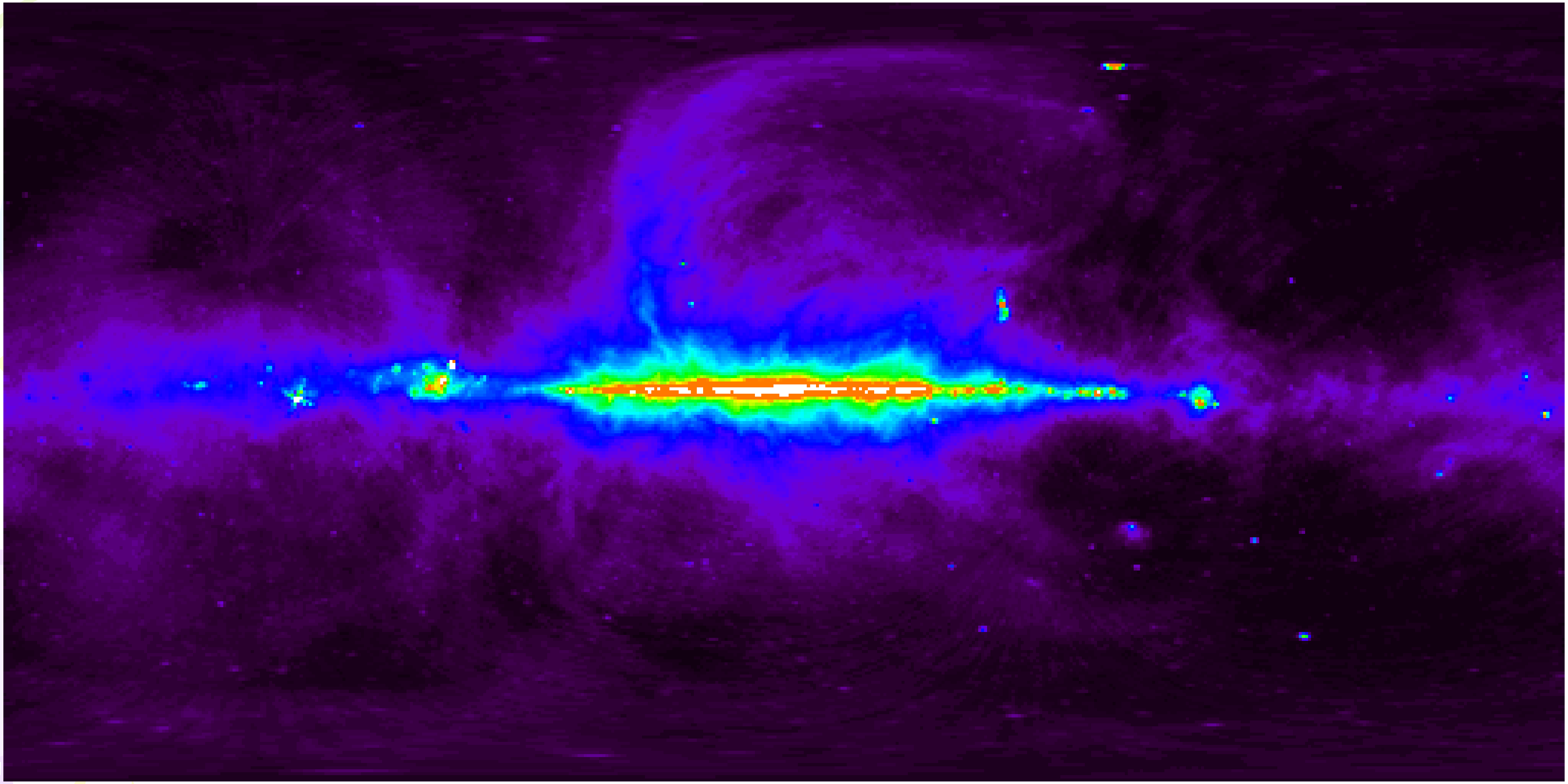
The SEFD gives $T_a = \dfrac{S\eta A}{2k}, (\eta A = A_e)$

From the system noise, T_N, can calculate bandwidth $(\Delta\nu)$ and integration time (τ) needed: $\sigma = \dfrac{T_N}{\sqrt{\tau \cdot \Delta\nu}}$. Usually want $T_a > 5\sigma$

Must remember that sky noise also contributes to T_N

- First there is emission from space:
  - Diffuse emission from the Galaxy
  - Emission from the source itself
  - The 2.7 K background
- 2.7 K is usually insignificant
- Galactic emission important at low frequencies (dominant noise source for $\nu < 200$ MHz)

The sky at 408 MHz



The atmosphere has an effect at short λ (<10 cm)

$$1.\ \text{Part of signal absorbed}: S' = Se^{-\tau}$$

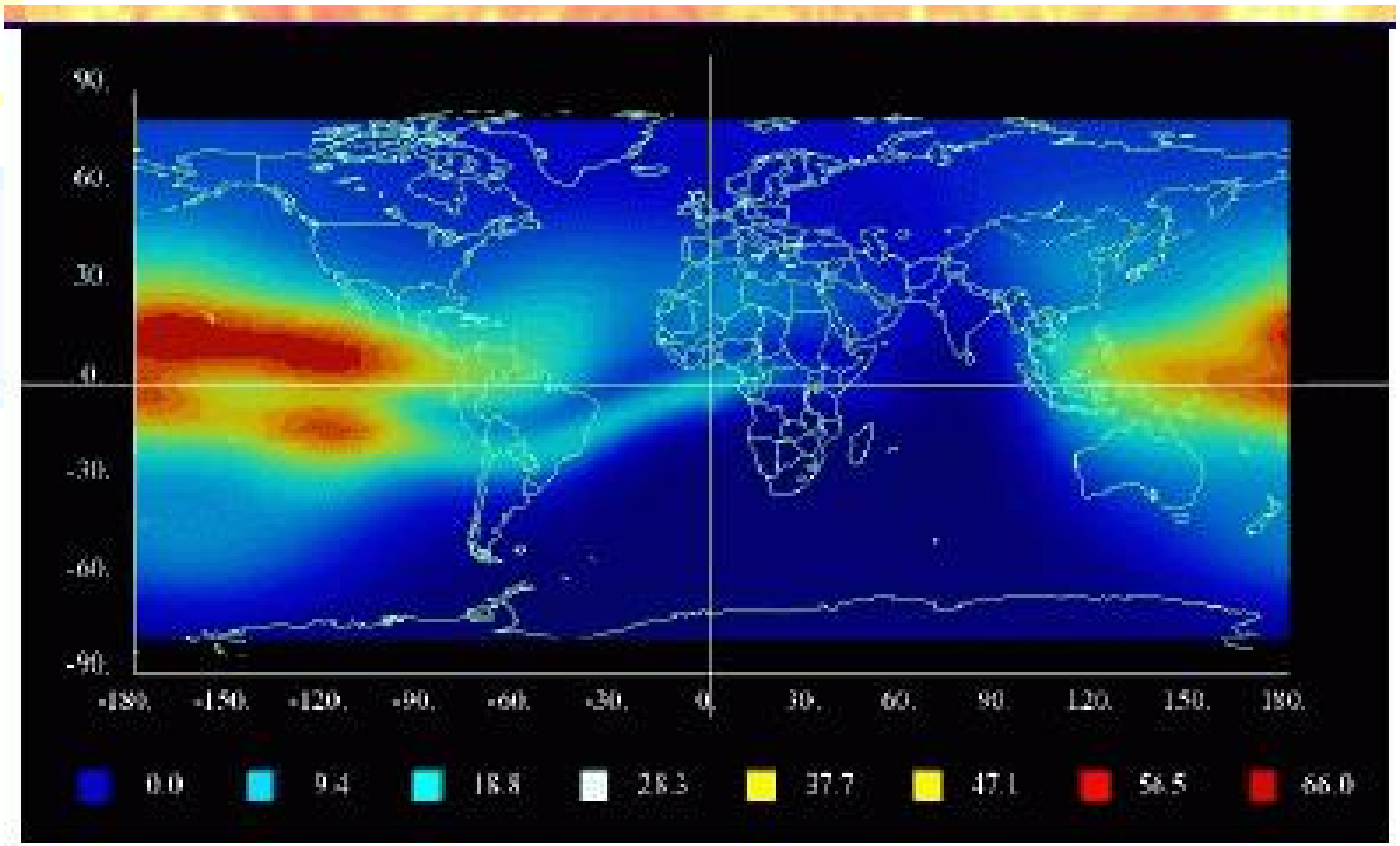
$$2.\ \text{More important, sky emission will}$$

$$\text{be picked up}: T' = T_{\text{sky}}(1 - e^{-\tau})$$

$$\text{Example: for } \tau = 0.1,\ S \text{ will be reduced}$$

$$\text{by } 10\%, \text{ and } T_N \text{ will increase by} \approx 25\ \text{K}$$

And at long wavelengths (>10 m), role of ionosphere



Rayleigh distance (or "near field" and "far field")

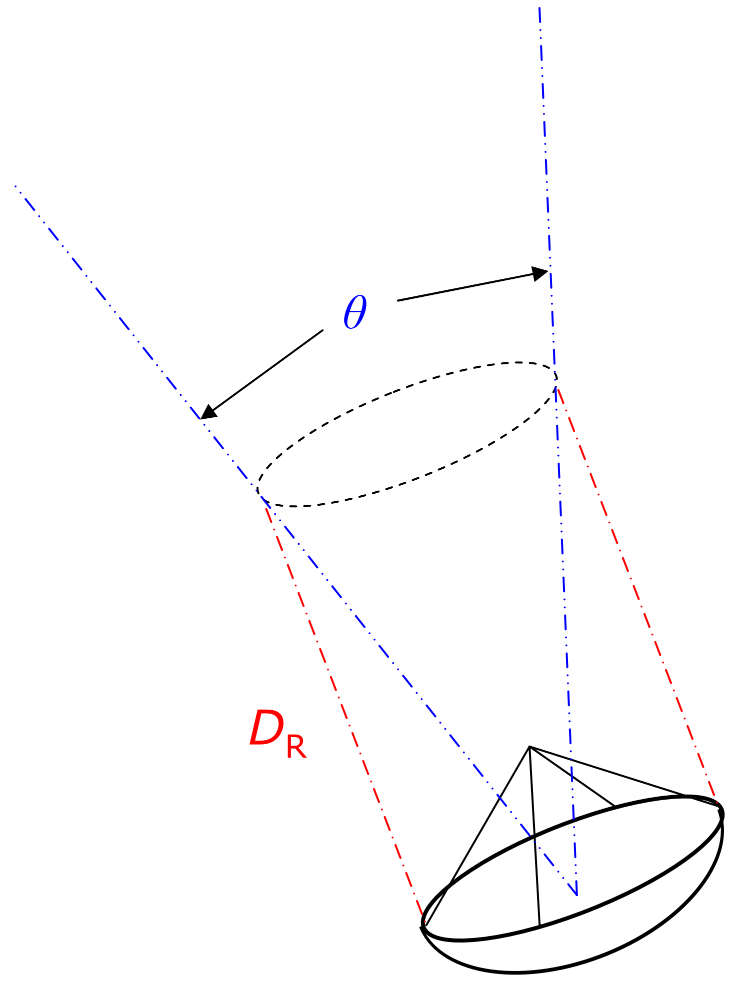
$$\theta \approx \frac{\lambda}{D}; \ \& \ D_R \approx \frac{D}{\theta} \approx \frac{D^2}{\lambda}$$

$\text{Example: } D = 25 \text{ m}, \lambda = 10 \text{ cm}$

$$\Rightarrow D_R \approx 6.25 \text{ km}$$

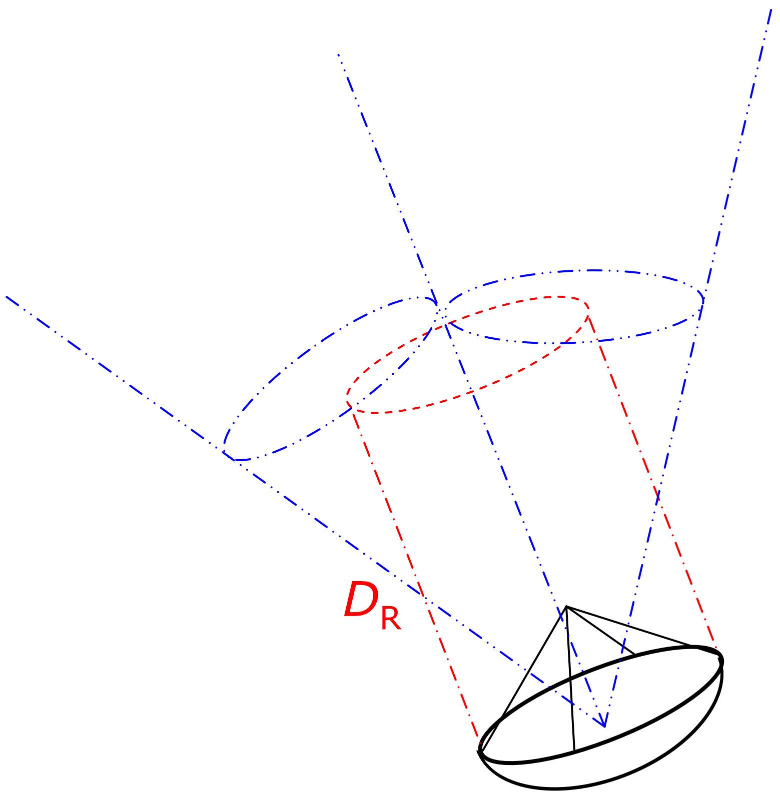
Only in *far field* (distance $> D_R$) do you have a true beam.

$\therefore$ Source distance $>> D_R$ (this is sometimes a problem with planets at short wavelengths). Also problem when measuring with transmitter.



At short wavelengths, can put 2 feeds in one dish

- These 2 beams pass through almost the same atmosphere.
- We can point one beam at source, other on empty sky.
- By switching between them, we can “switch out” sky signal.

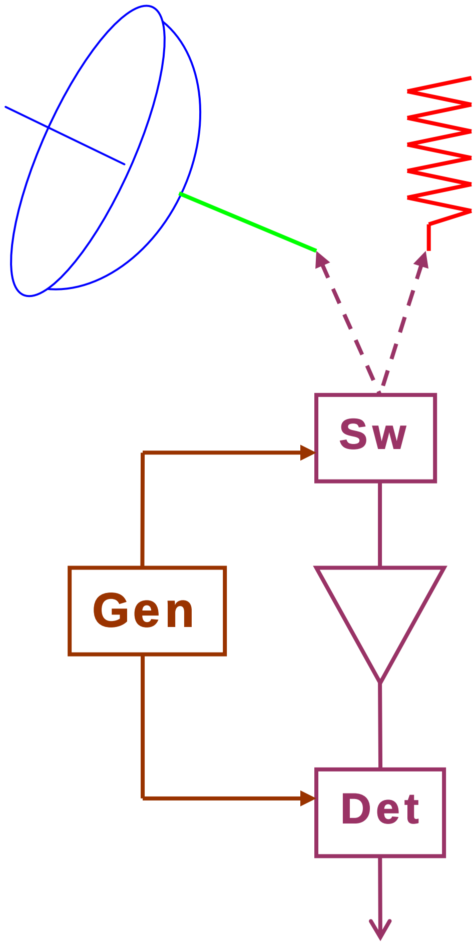


Having good sensitivity is useless if stability is poor

- Amplifiers with high gain tend to be less stable
- To keep output stable, often add feedback loop: automatic gain control (AGC)
- Physicist Robert Dicke invented technique: switch to reference noise source, to monitor receiver.

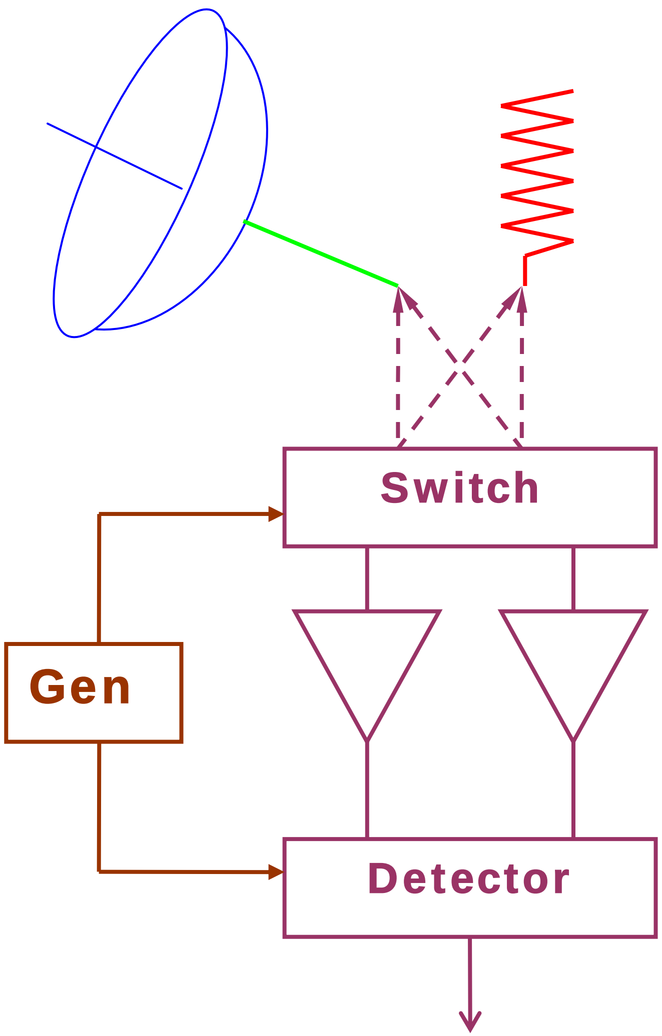
Example of a simple Dicke switch radio telescope

- Generate switching frequency, faster than system drift
- Demodulate at same frequency after detection
- Disadvantage is not all time spent on source: lose some observing time



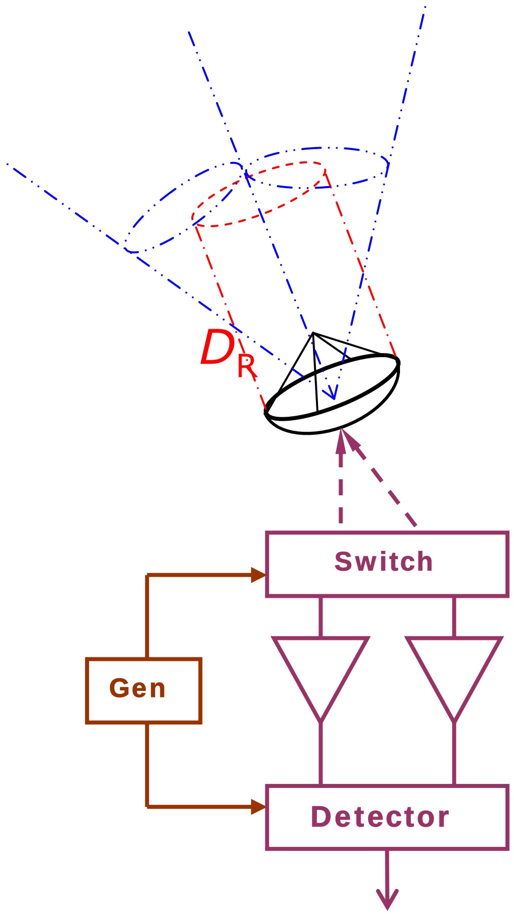
Avoid loss of observing time with two receivers

- Always observing sky and reference
- At end, average two difference signals
- Always need stable reference
- This system costs more (2 channels)

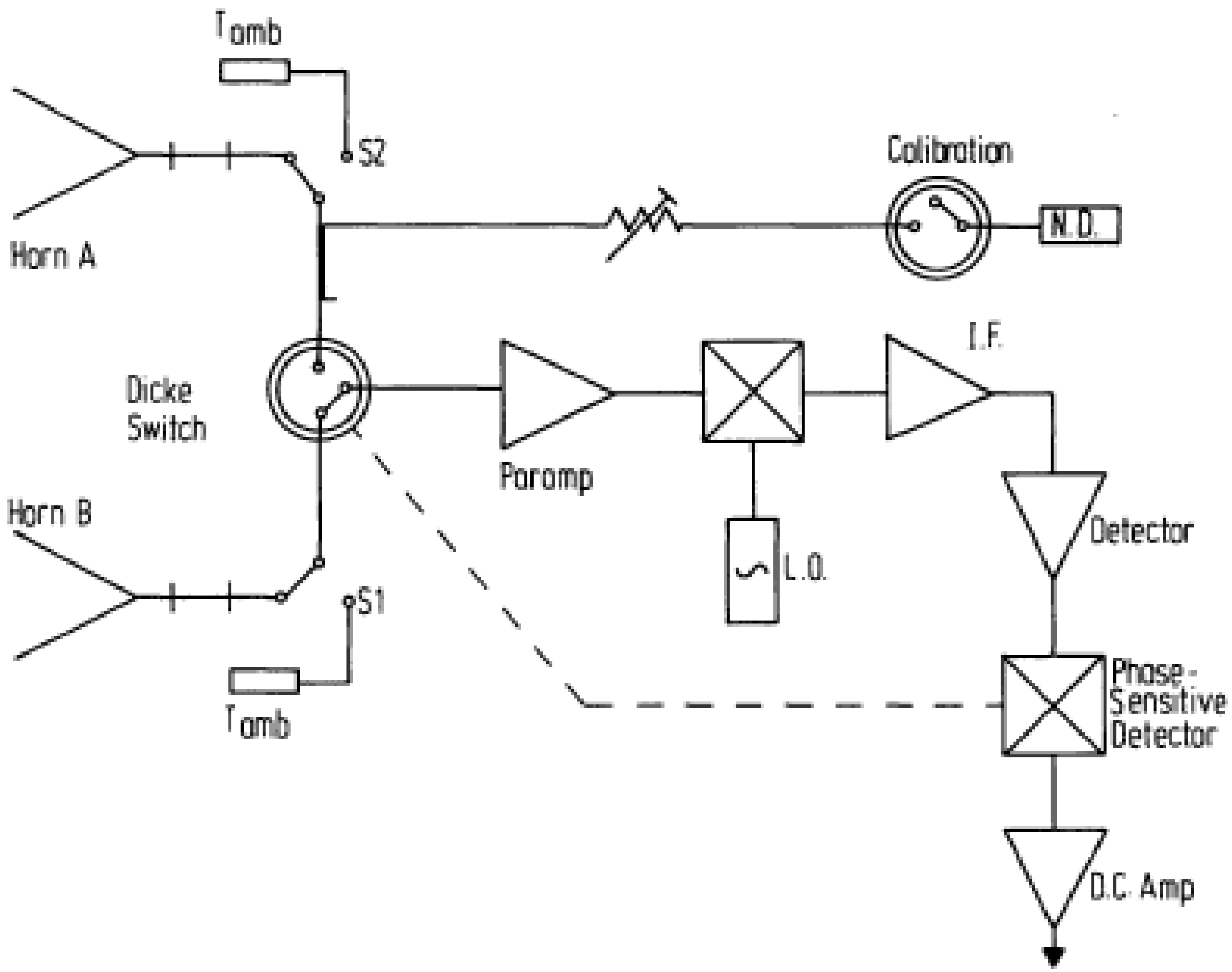


Dicke’s technique widely used, in different ways

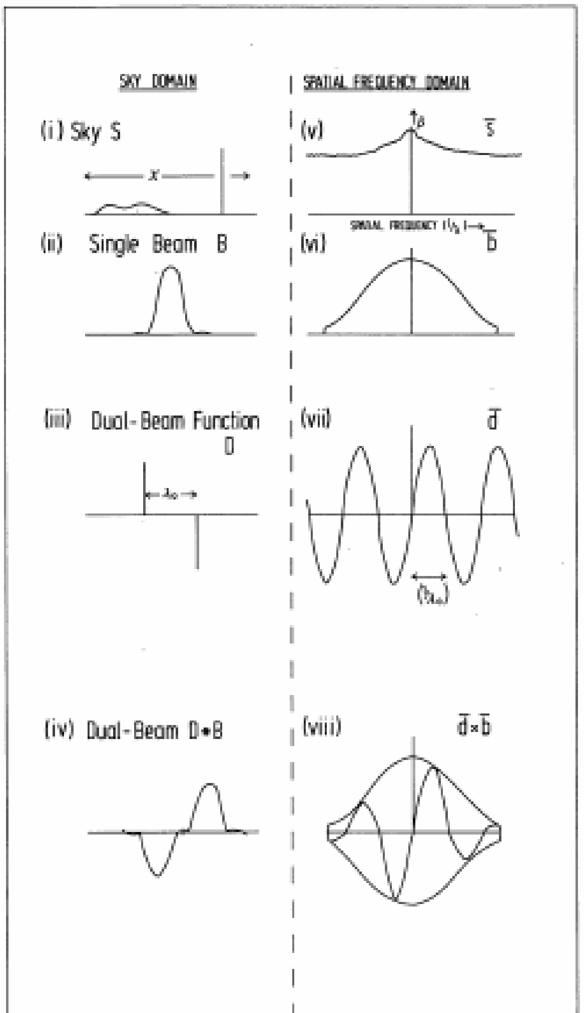
- For example, with two receivers, we can make two beams
- We can point one beam at source, other on empty sky.
- Using Dicke’s switch, one beam becomes reference – can “switch out” effect of atmosphere.

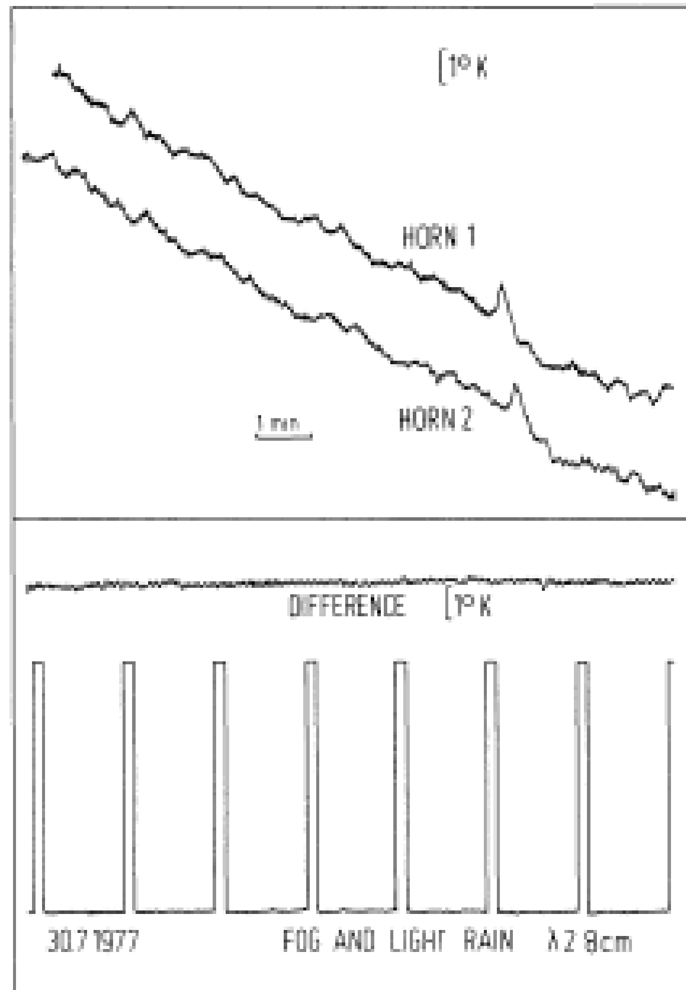


Effelsberg λ2.8 cm system (Emerson et al., 1979)

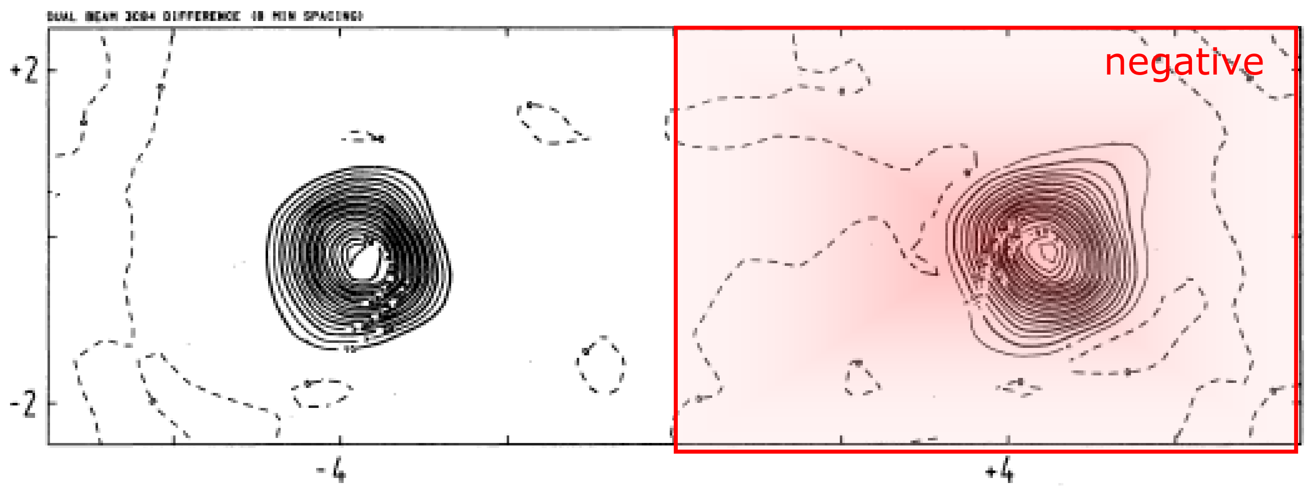


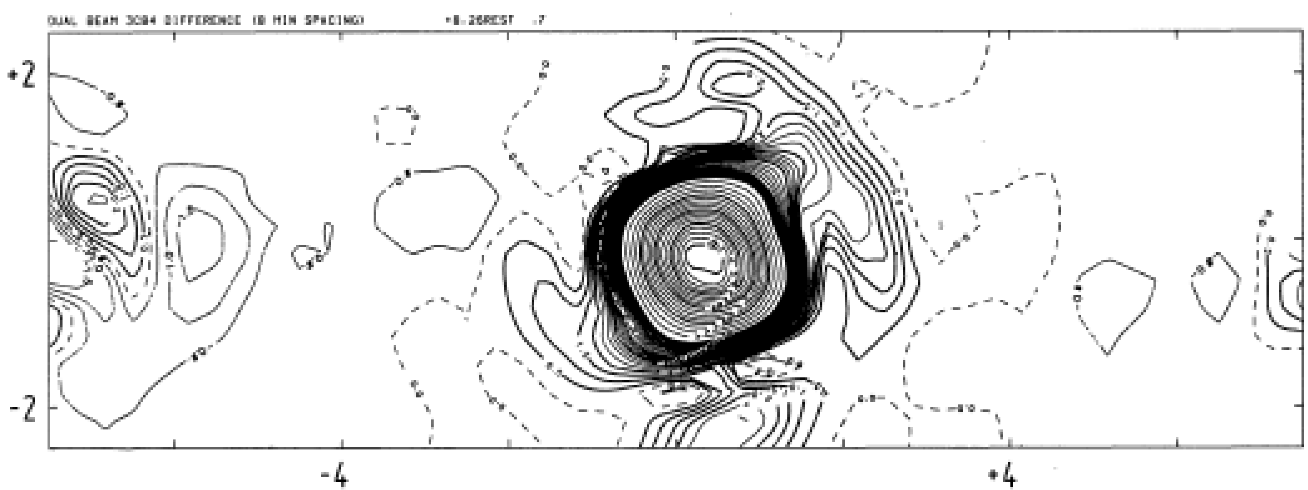
What dual-beam measures & example of data (in fog)





Observation of strong source 3C84: data & result

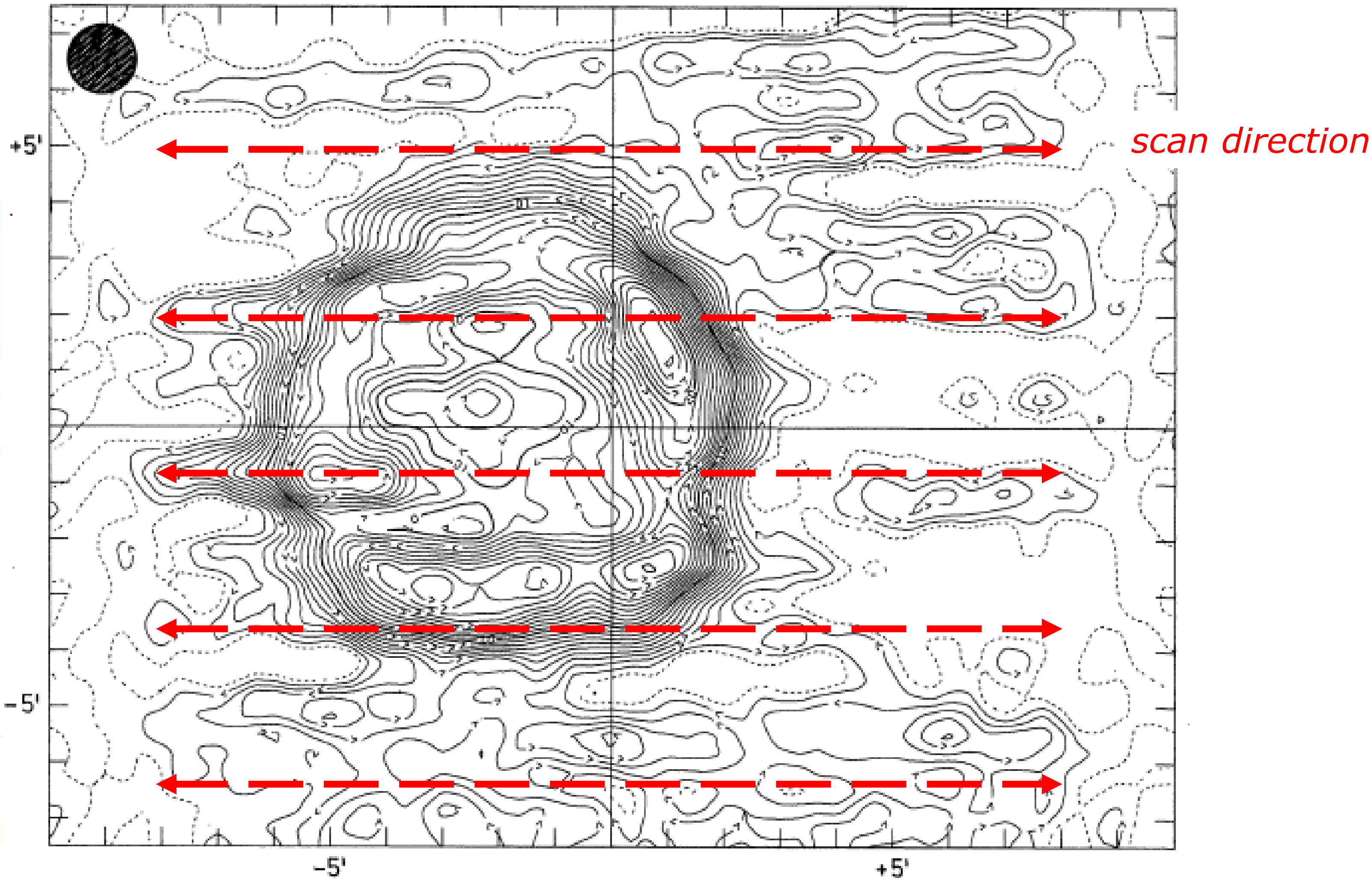




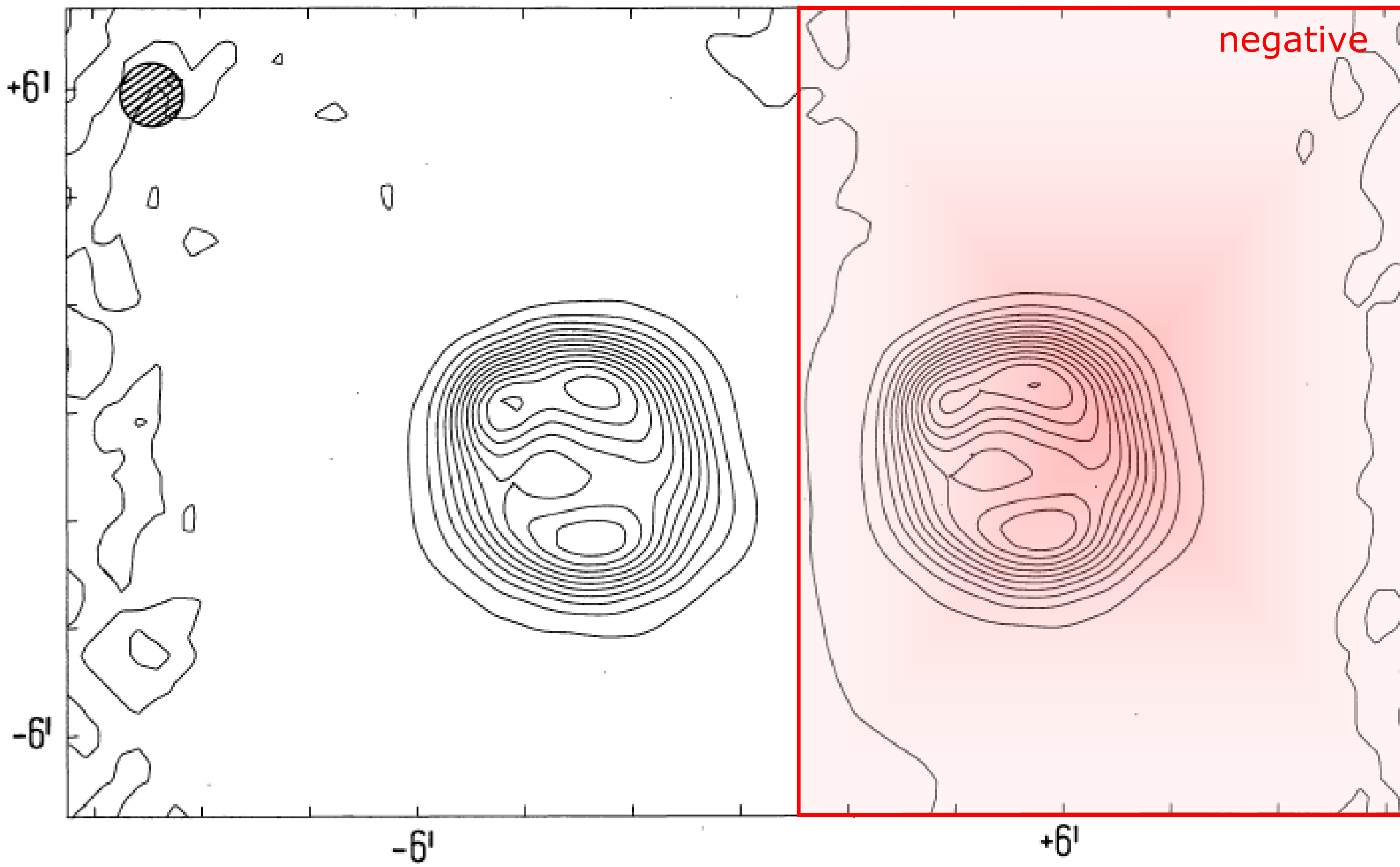
Technique can also be used for mapping extended sources

- For Effelsberg dish (100 m diameter) observing at λ=2.8 cm
- Rayleigh distance: $D_R \approx D^2/\lambda = 100^2/0.028 = 360$ km
- Troposphere (where water is) is at 2-3 km altitude, so should be same in both beams

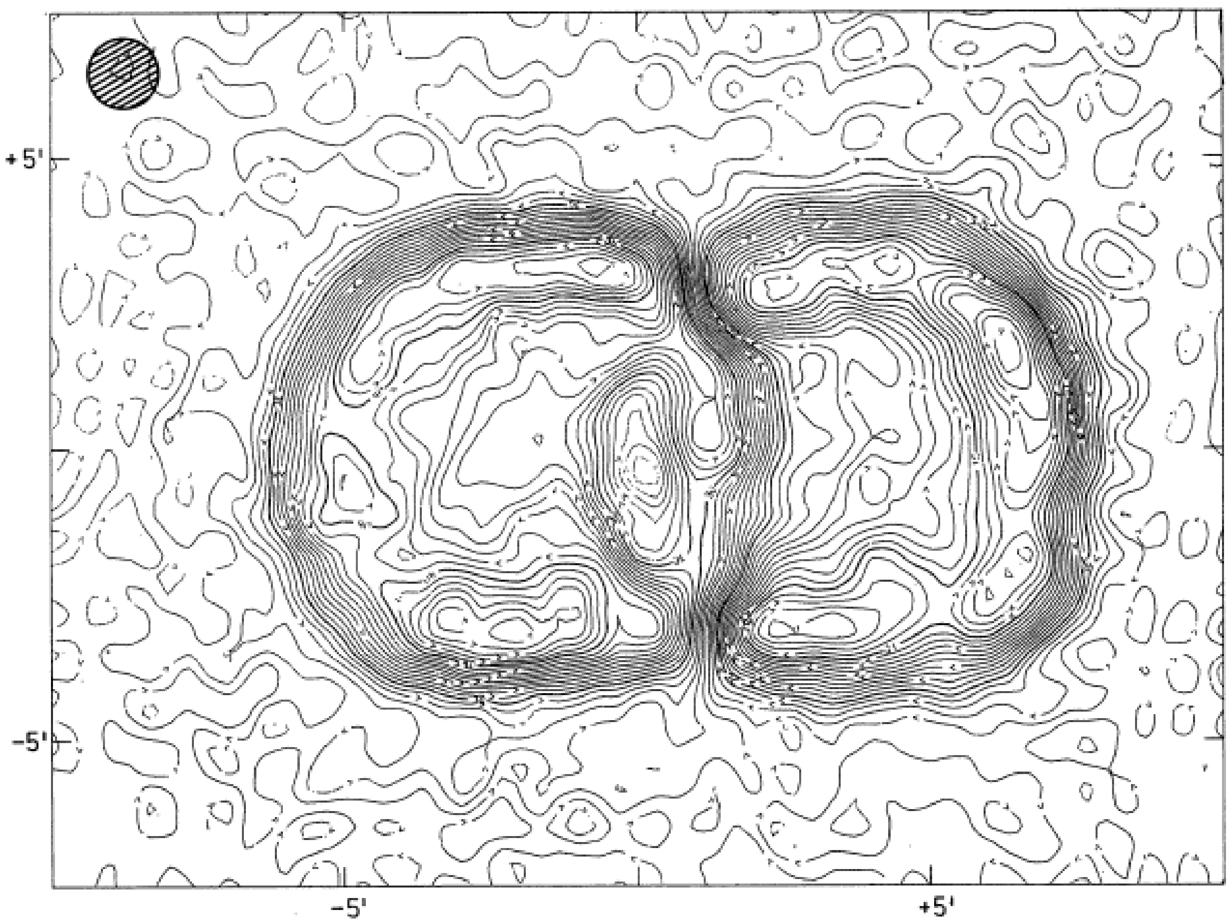
Single-beam map of 3C10, showing effects of atmosphere



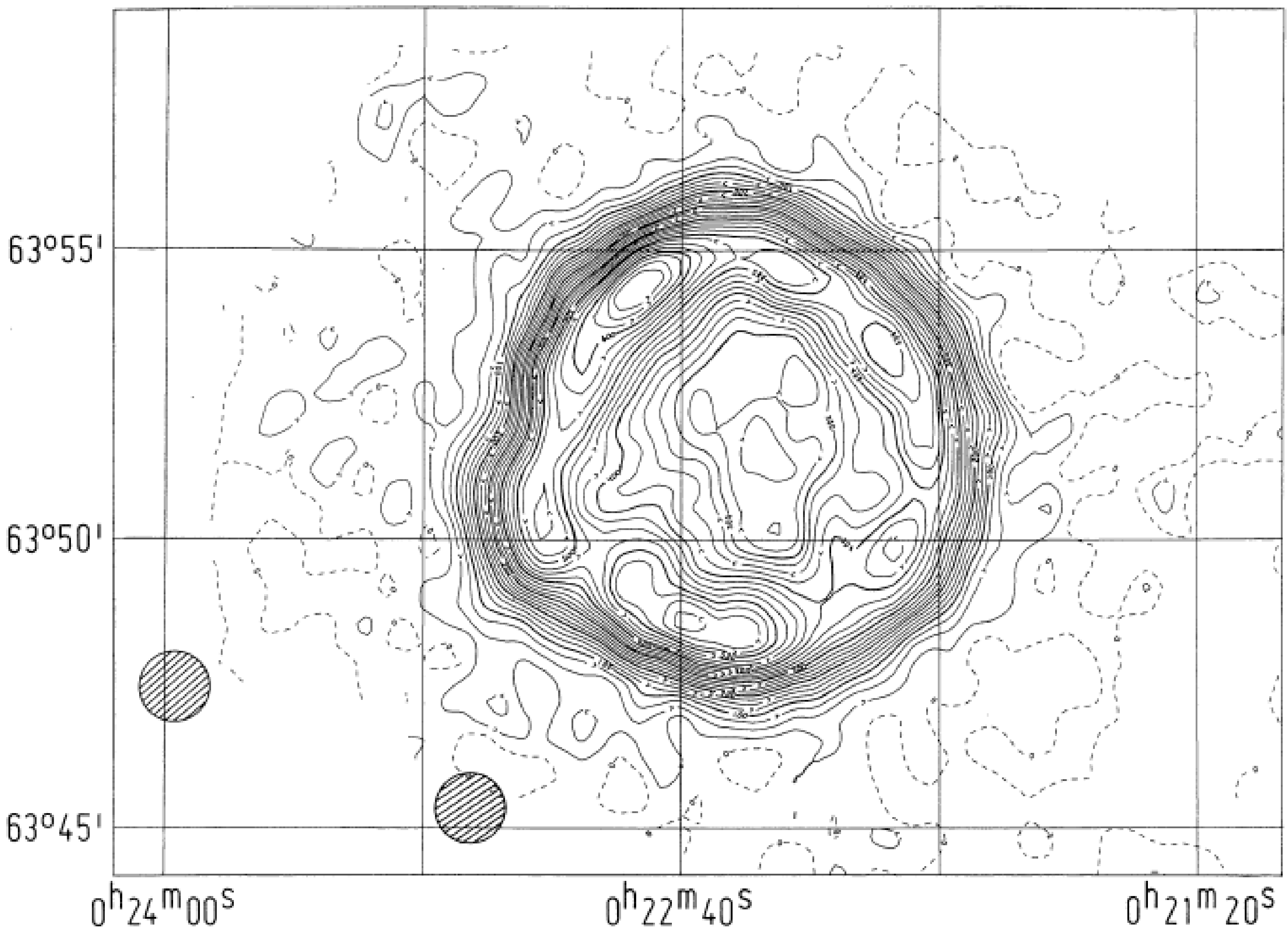
Cas A, beam separation = 8.2′ arc: 2 images well separated



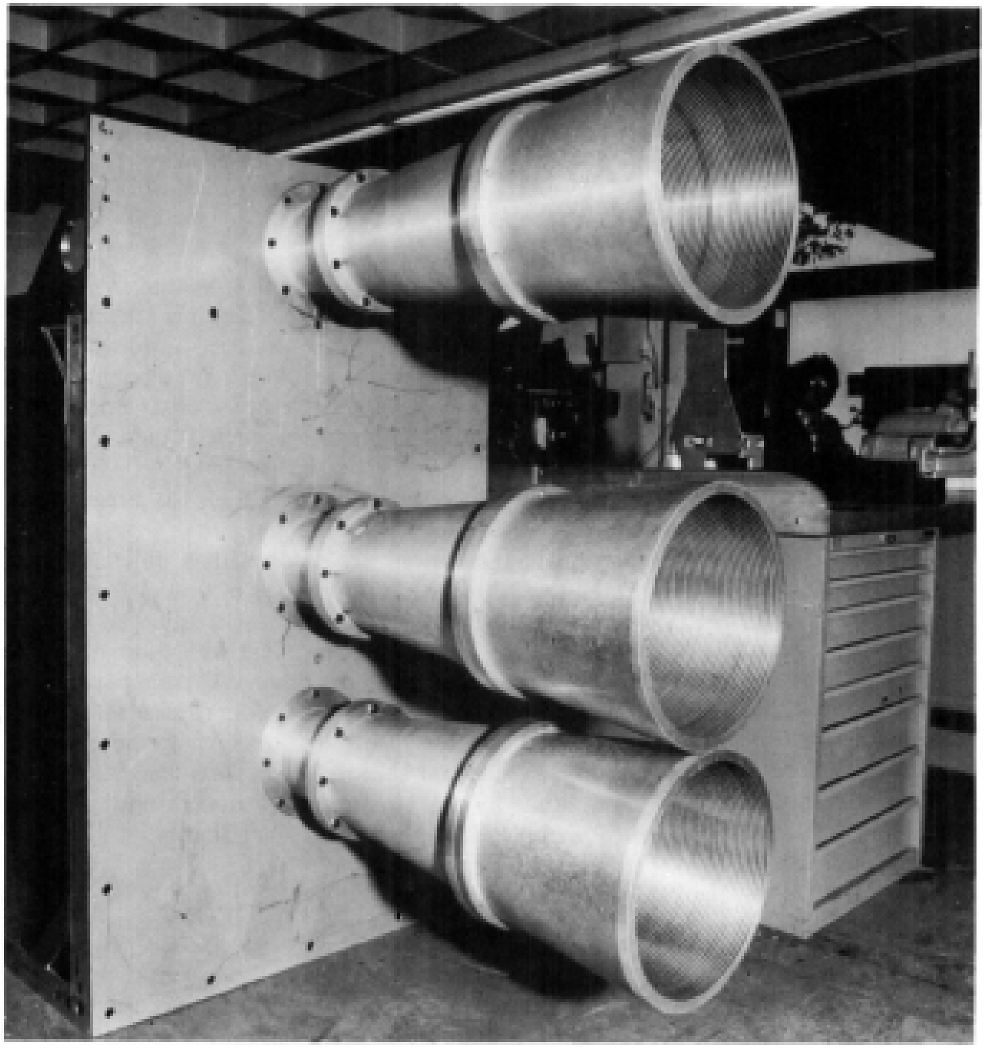
Images not always separated: 3C10, 5.5’ arc beam distance



3C10, final map separates and averages two images



Triple-horn system: 3 beams are even better



Types of parabola feed systems – 1. prime focus

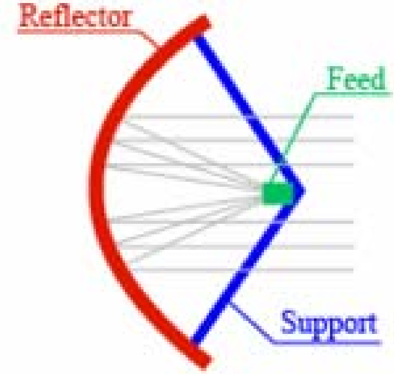
- Advantages are simplicity, cost, low blockage, wind loading, easy illumination
- Disadvantages are spillover, lower efficiency, space available

2. Secondary focus: Cassegrain or Gregorian

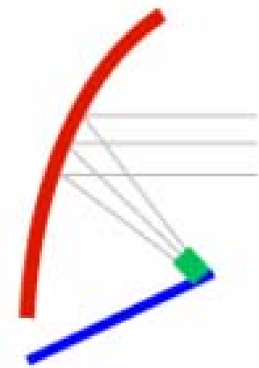
[Gregorian: concave mirror.]

- Advantages are lower spillover, better illumination (also “shaped”), more space.
- Disadvantages are wind loading, long λ feed (are short λ dishes), cost.

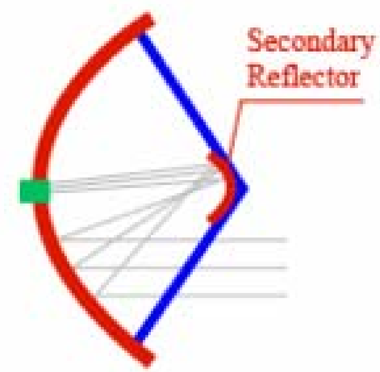
Some of the basic types of reflector and feed system combinations used with radio telescopes



Parabolic

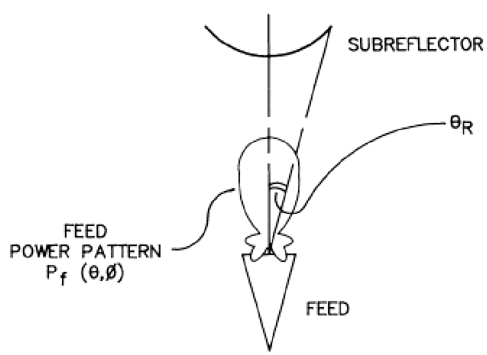


Off-center



Cassegrain

Spillover



Cassegrain and Gregorian reflector systems illustrated

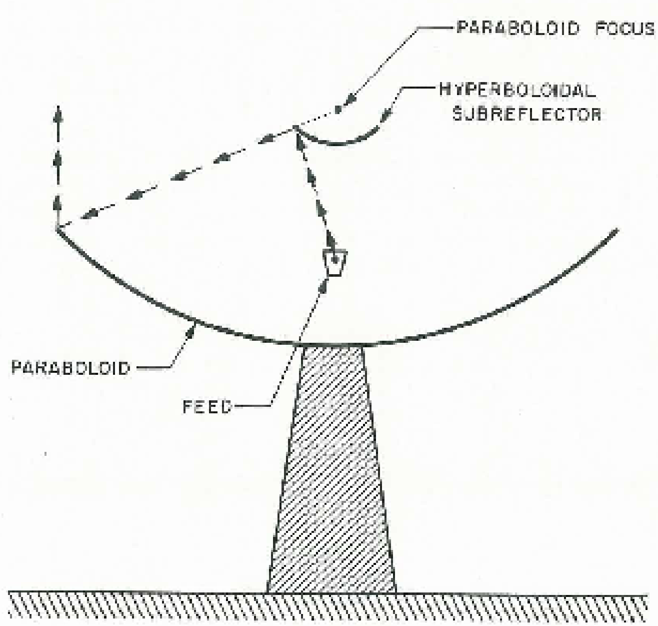


FIG. 15. Geometry of Cassegrain antenna.

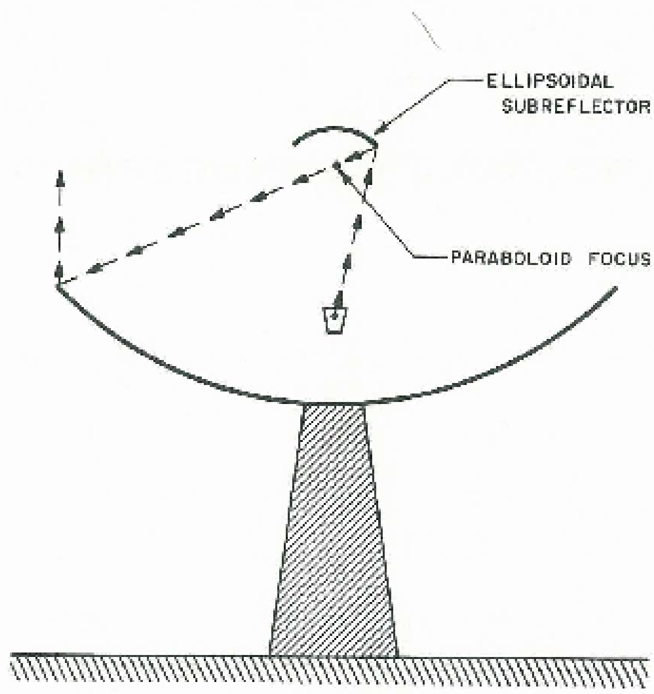
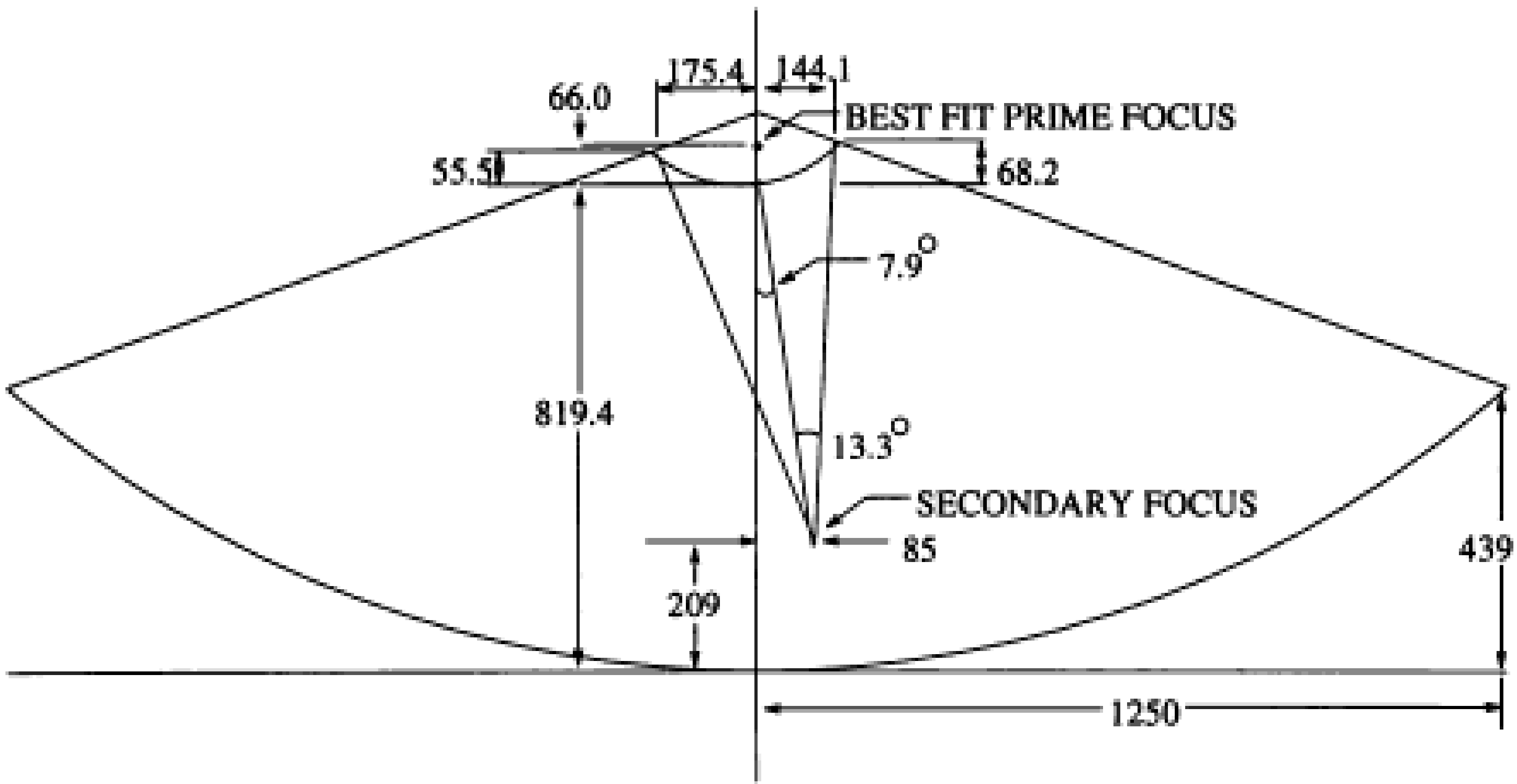


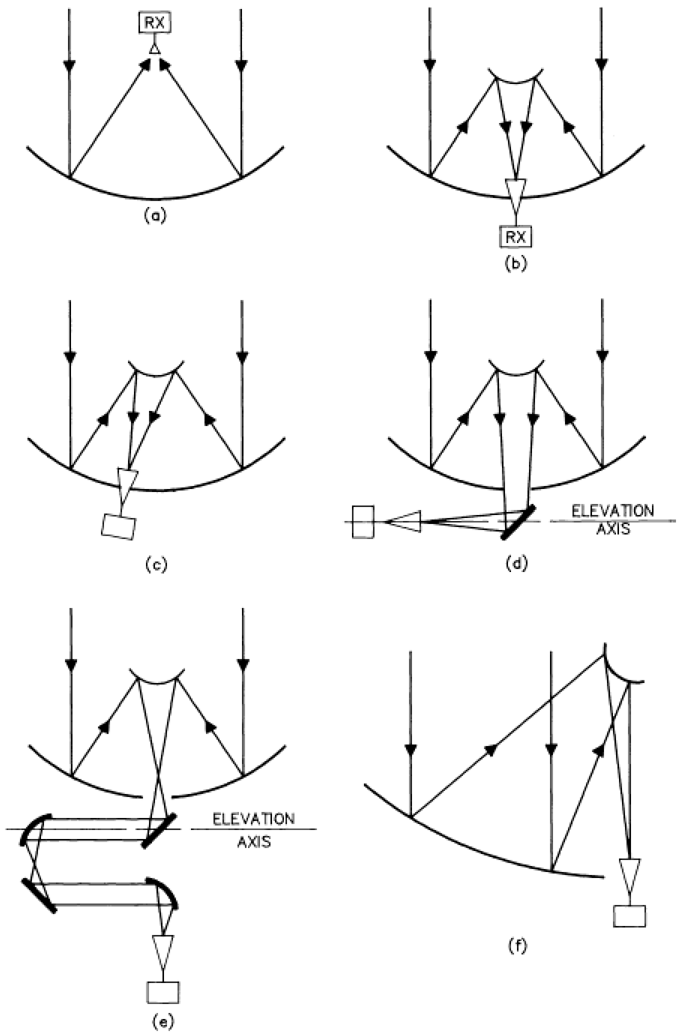
FIG. 16. Geometry of Gregorian antenna.

Sub-reflector and secondary off-axis foci in VLBA dish

Sketch of VLBA configuration

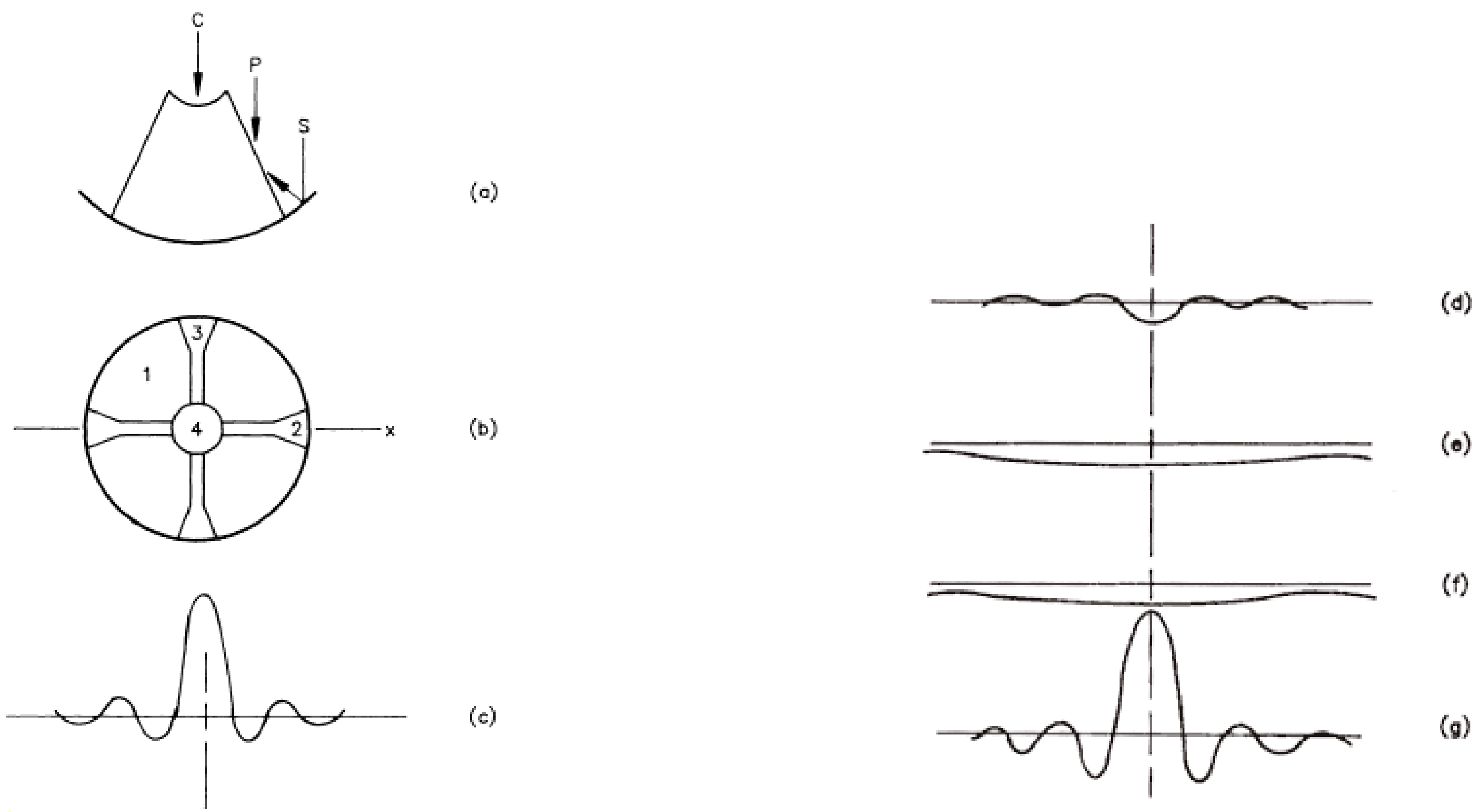


Location of focus: many variations are possible

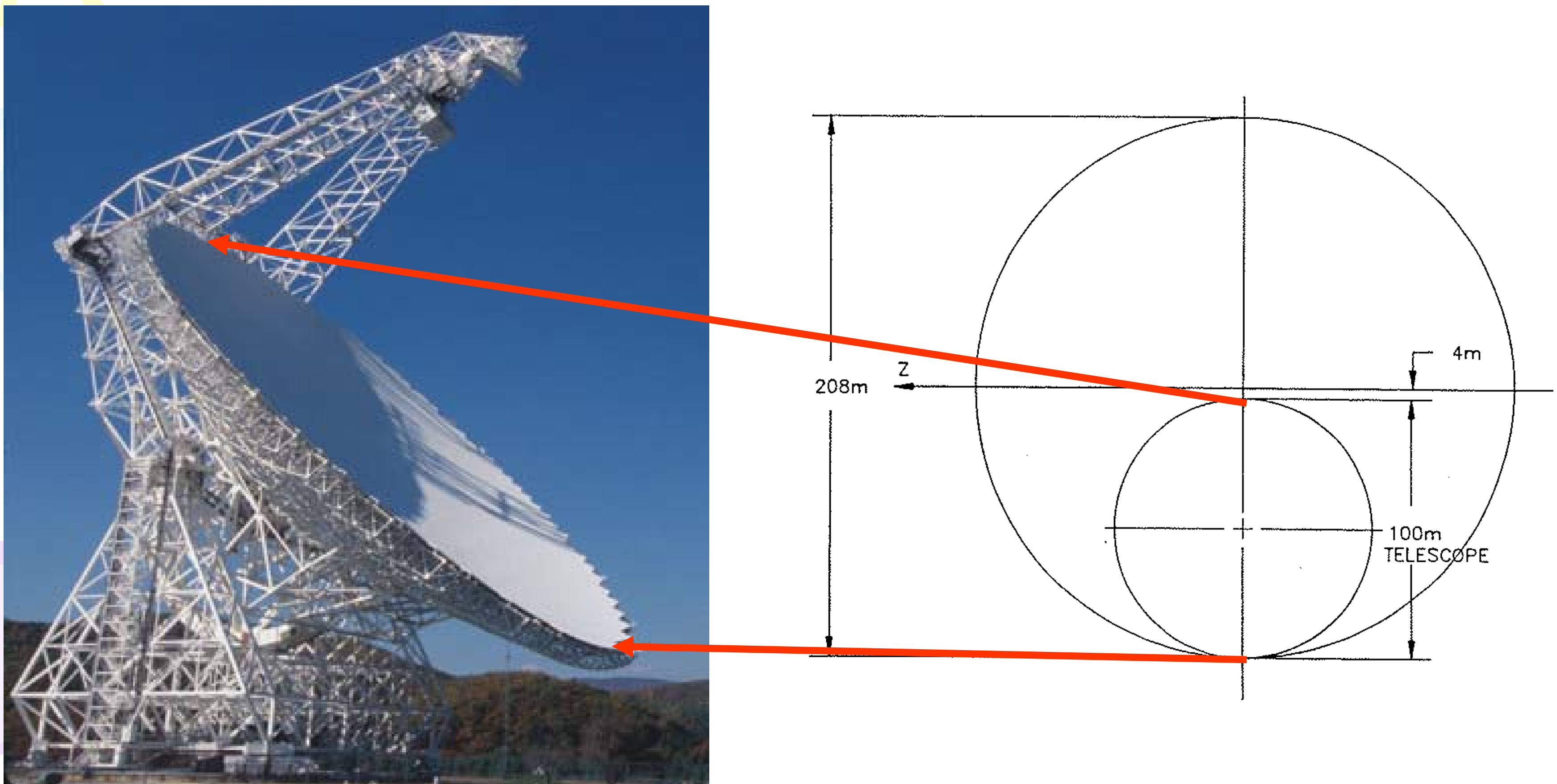


a. Prime focus
b. Cassegrain
c. Off-axis Cassegrain
d. Naysmith
e. Beam waveguide
f. Offset Cassegrain

The effects of blockage (a,b) on beam (c) can be modelled (d-g)



Offset secondary: no blockage or standing waves (but expensive)

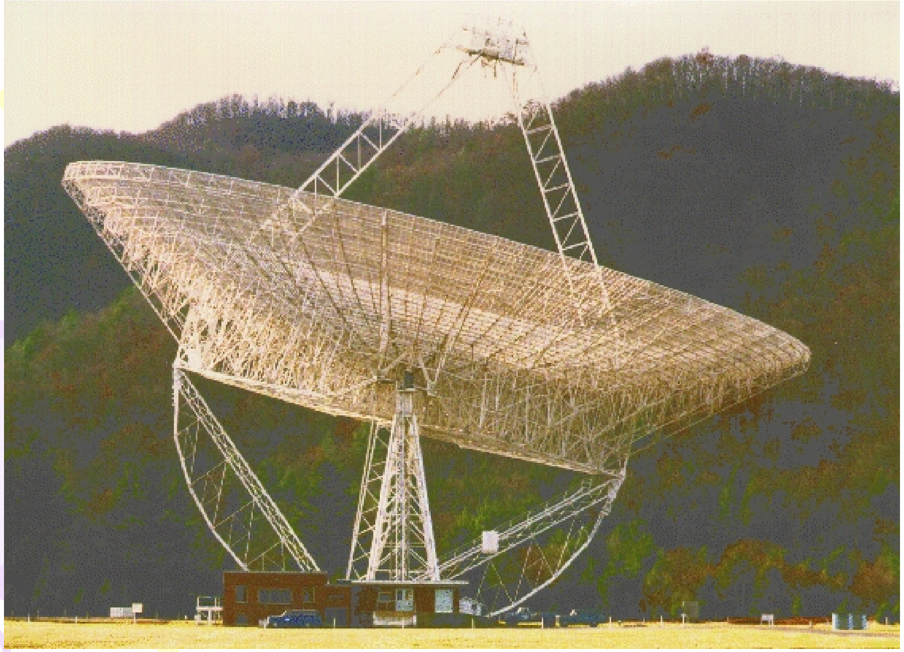


Building large surface and moving it is challenging and expensive

Greenbank 91 m telescope: meridian transit

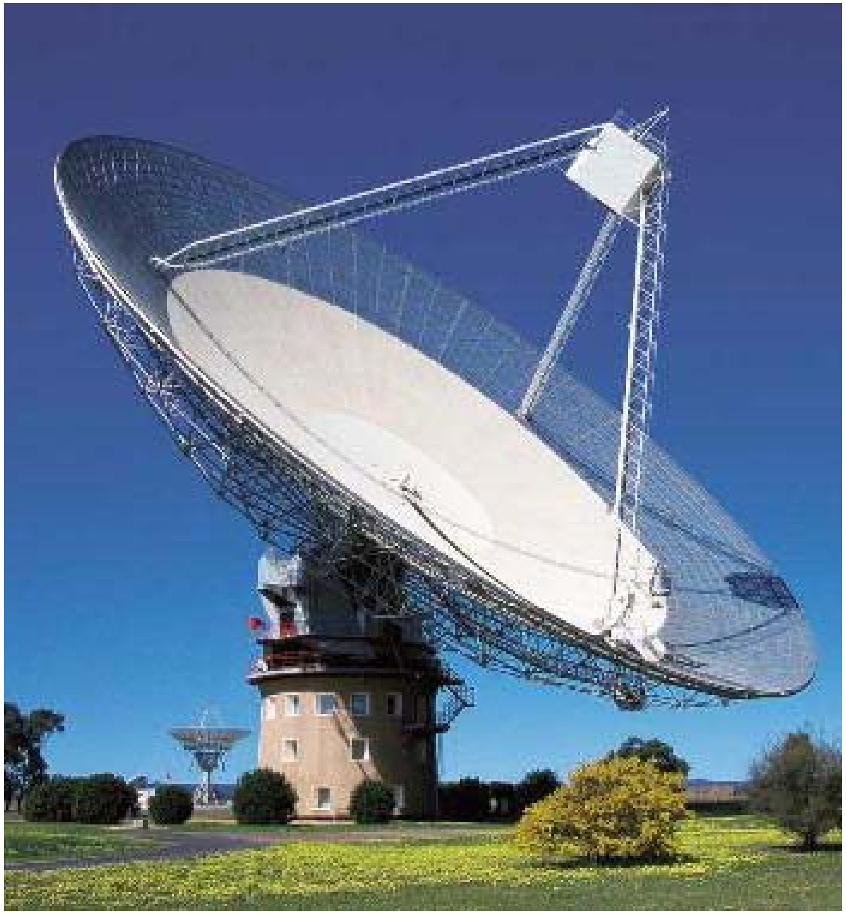
Inexpensive, “temporary” structure, but very useful as a survey instrument.

Unfortunately, one night…



Solid surface? Or mesh? Mainly question of cost

- Mesh can be good reflector, if holes have size « λ
- Mesh lighter, less wind loading
- Fixes shortest λ, some leakage
- Some dishes use mesh & solid

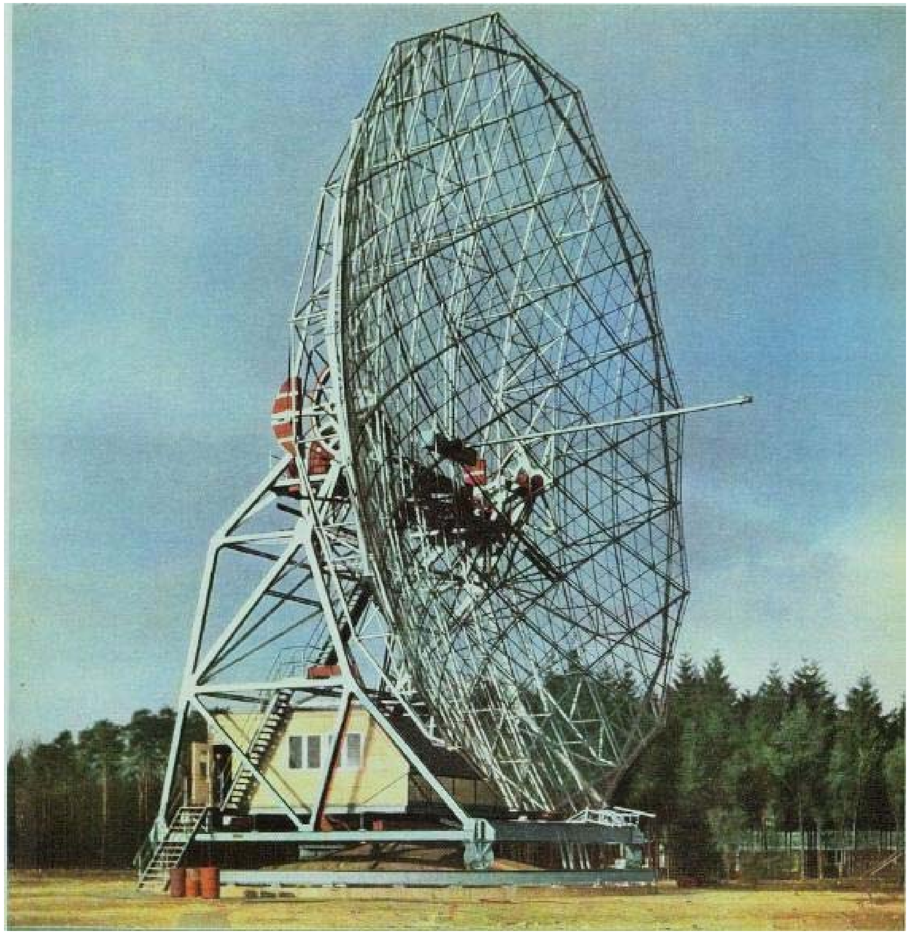


Reflector surface also affects antenna efficiency

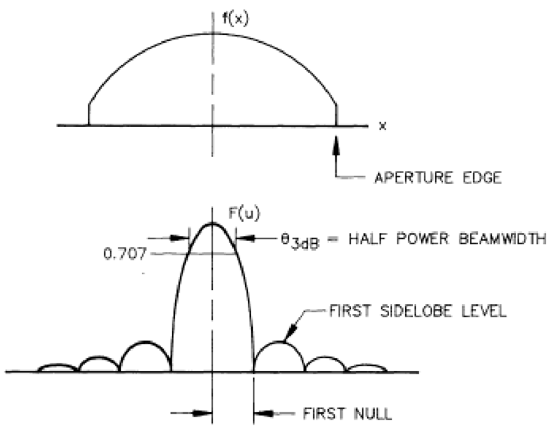
As λ shortens to near surface limit:

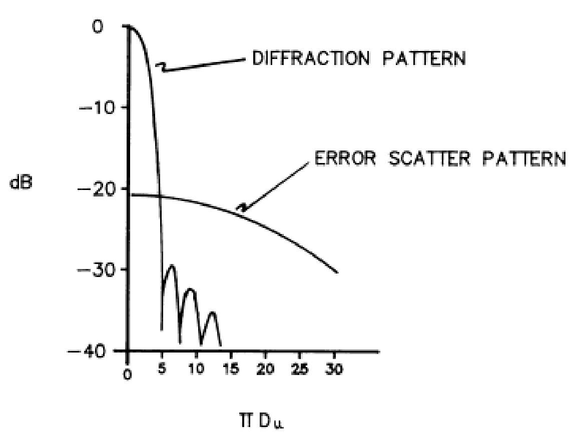
- because of mesh size, and/or
- because of surface irregularities

Efficiency, η, will decrease, lowering A_e (advantage of solid surface: no leakage)



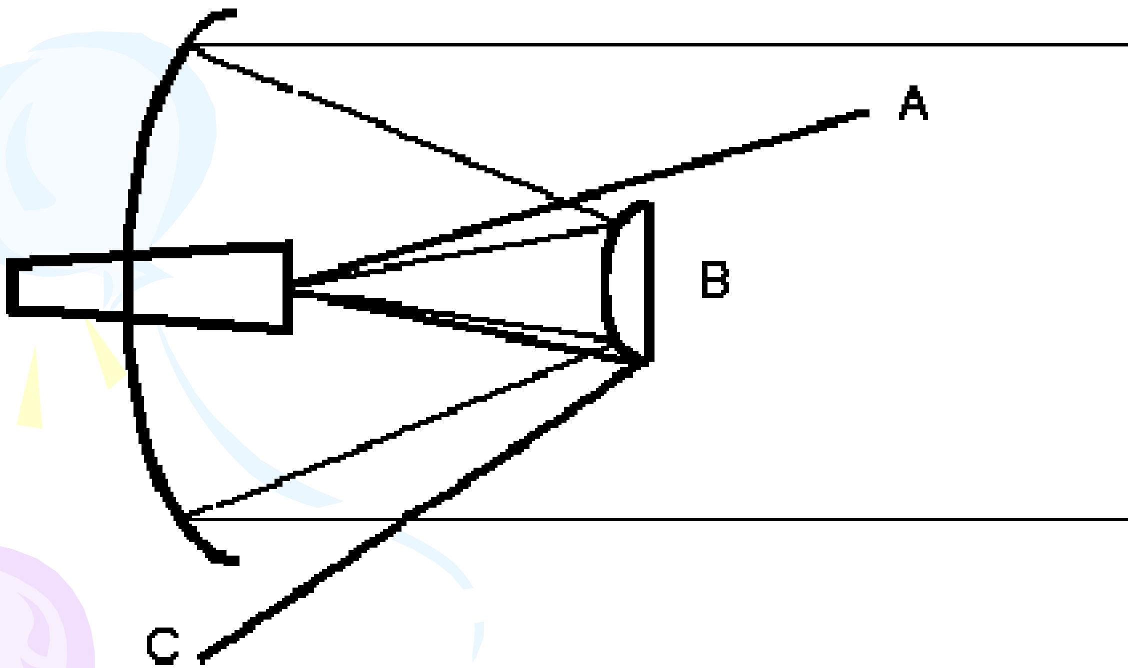
Surface irregularities give scatter sidelobes





- The beam pattern is determined by FT of illumination
- Irregularities of size λ/16 produce error scatter
- True beam pattern is sum of diffraction + scatter patterns

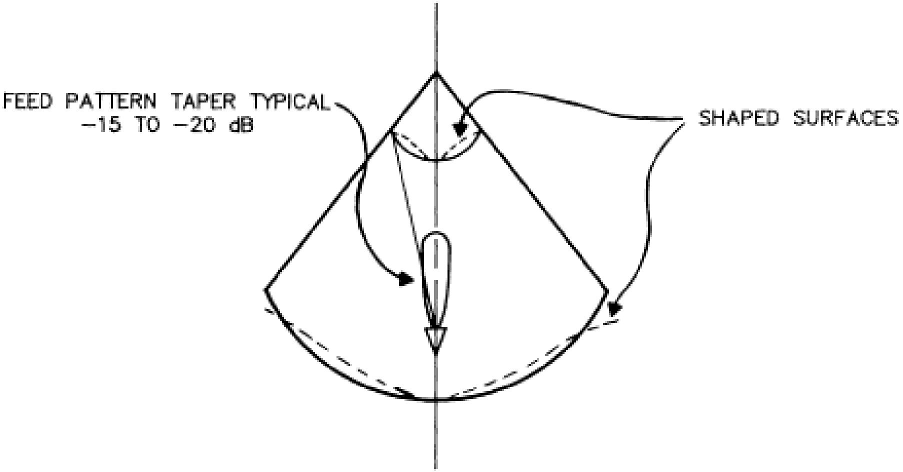
Why is A_e always < A_{phy} in (parabolic) reflectors?



Main factors:

- Spillover
- Blockage of primary
- Surface imperfections
- Ohmic-losses
- Non-uniform illumination

We can use "shaped" dish to increase A_e (VLBA)



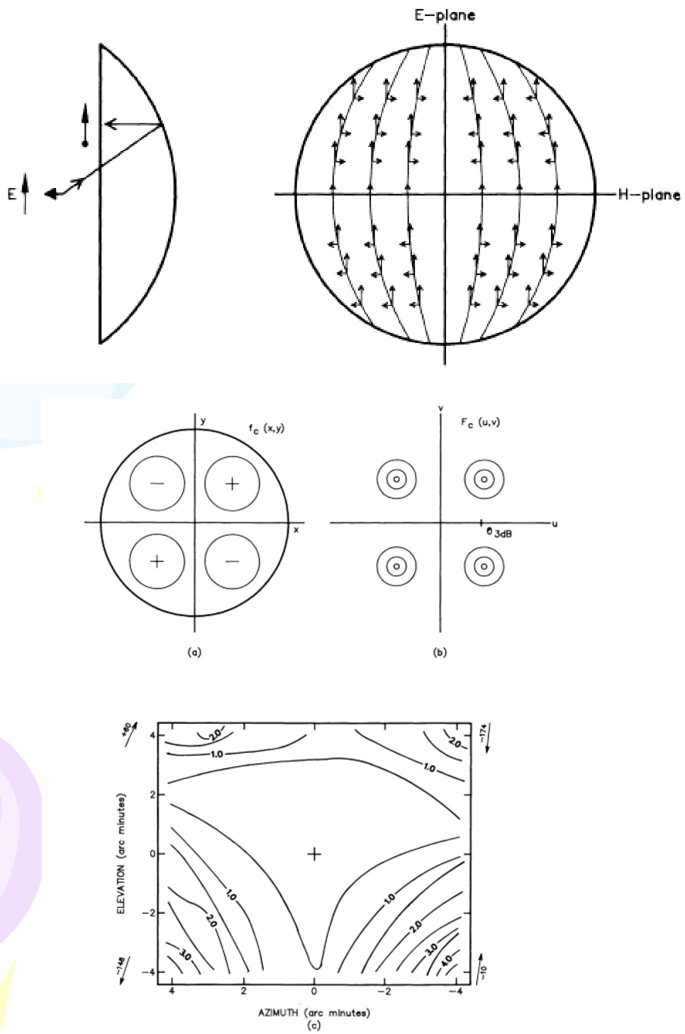
With one polarization, we lose half of the signal!

Most radio sources are weakly polarized (<10%).

To receive all of the emission, need to use 2 receivers.

Feeds should pick up orthogonal polarizations.

What is polarization response of parabolic dish?



- The *E*-field induced in the dish shows “barrel” distortion
- This gives unwanted components of *E*
- These are symmetrical and cancel on-axis
- But off-axis, we see apparently polarized emission

Next lecture we will look at interferometers